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New Findings from Terrorism Data:

Dirichlet Process Random Effects Models for Latent Groups

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Introduction
Terrorism Data

- The analysis of data on terrorists and terrorist attacks is difficult.

- Typical data are
  - Observed public events
  - Not including failed attacks
- Classified government information
- Terrorists seek to strategically hide information

Introduction
Terrorism Data

- Data collection can even be physically dangerous for the researcher

- Terrorism is an important problem
  - It affects personal safety
  - Internal government policies
  - Public perception
  - Relations between nations

Introduction
Overview of the Talk

▶ Background about terrorism data sets	Problems with the data
▶ Logistic Random Effects Models	An Introduction to Modelling Random Effects
▶ Fitting the Models	Markov Chain Monte Carlo
▶ Analysis of a Terrorism Data Set	What the Covariates Explain
▶ Conclusions	What We Learned

Background On Terrorism Data
Types of Data Available

- Most of the datasets focus on *incidents*
- Data from an observed violent attack and covariates such as
  - Responsible group
  - Target characteristics
  - The extent of casualties and damage.
- Humans in terrorist networks conceal their identities and intentions
- Therefore there is a lack of informative covariates

Background On Terrorism Data
Major Databases

- University of Maryland (START)
- US Homeland Security Agency
- International Terrorism: Attributes of Terrorist Events (ITERATE)
  - Records transnational terrorist incidents
- International Policy Institute for Counter-Terrorism in Herzlia, Israel
  - Detailed online database of terrorist attacks in Israel
- The Global Terrorism Database (GTD)
  - Information on global terrorist events starting from 1970
  - We used this one

Background On Terrorism Data
Previous Findings

- Extremist groups often $\uparrow$ terrorist activity after government concessions
  - Anecdotal evidence rather than statistical data analysis

- Statistical models try to forecast the occurrence of terrorists incidents
  - Limited results

- Networks of terrorist and terrorist organizations
  - Tend to be cellular and independent
  - Rather than hierarchical and connected

Background On Terrorism Data
Data Problems

- Not much success in building standard regression models
  - The data are, in general, poorly measured
  - Categorical variables with large variability

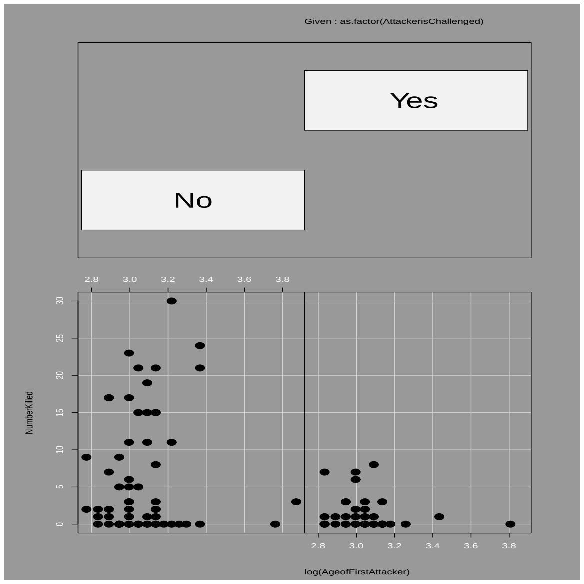
- Huge Problem: The terrorists under study
  - Are deliberately trying to prevent accurate data from being collected

- The statistician has a difficult task in creating meaningful models.

Background On Terrorism Data

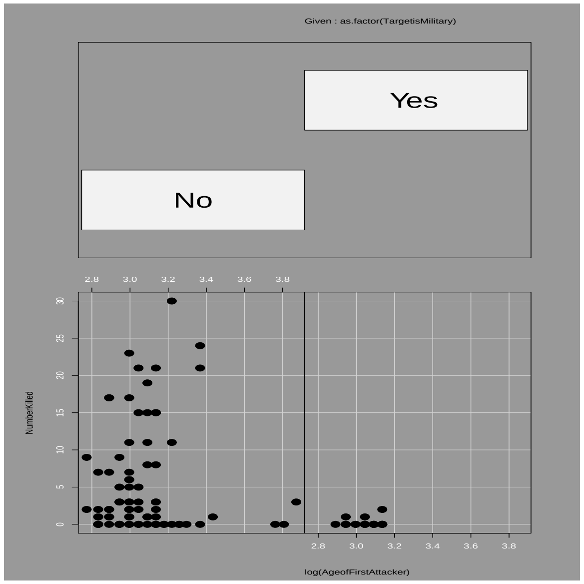
Data Quality Example: Attacks in Israel

Attacker is Challenged



- Y-axis = Number of Casualties

Target is Military

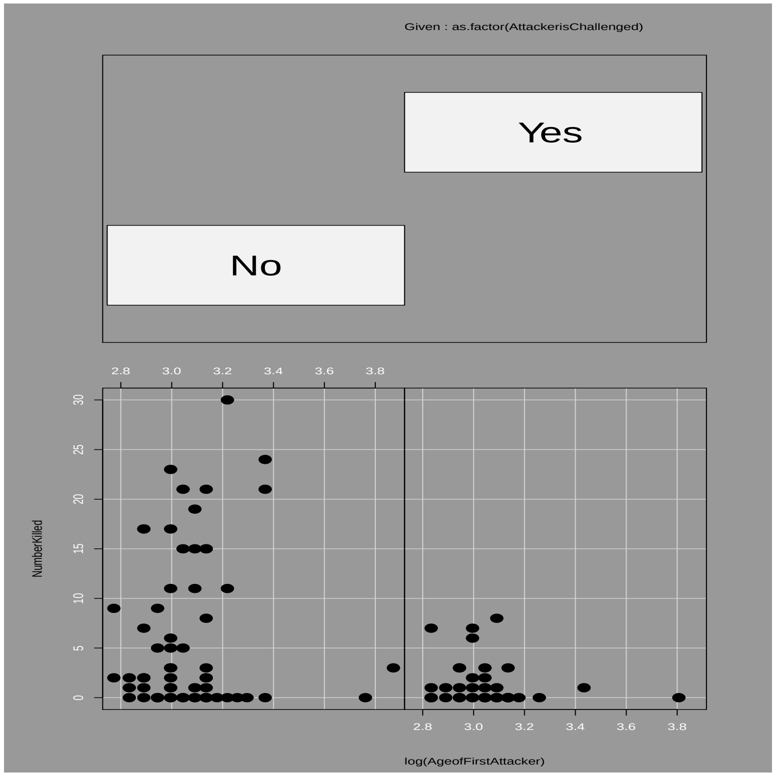


- X-axis = Age of Attacker

- Consider some details

Background On Terrorism Data
Attacks in Israel – Attacker is Challenged

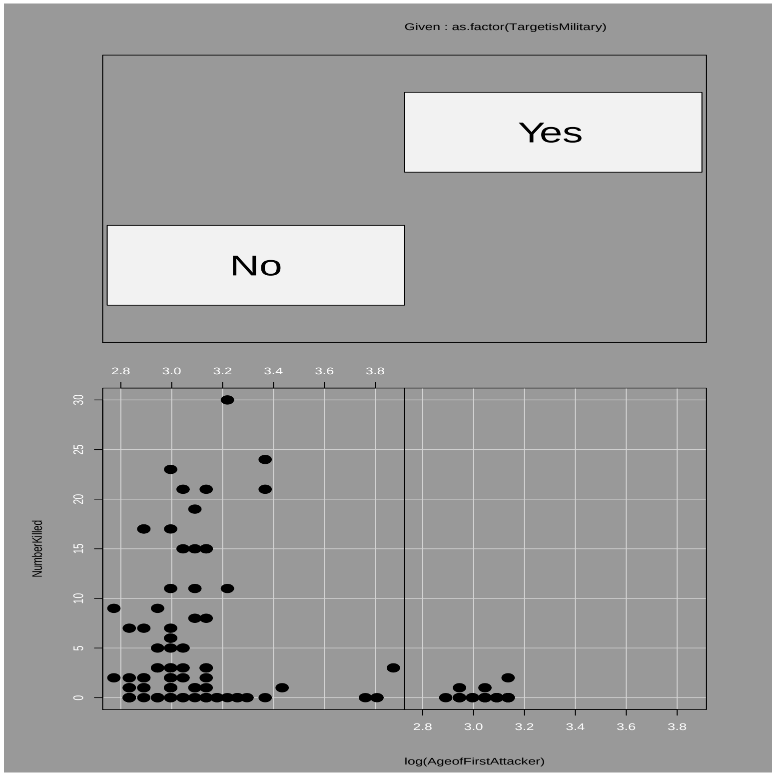
Attacker is Challenged



- Difference in the distribution of fatalities between the plots.
- The attack is less deadly if the attacker is challenged

Background On Terrorism Data
Attacks in Israel – Target is Military

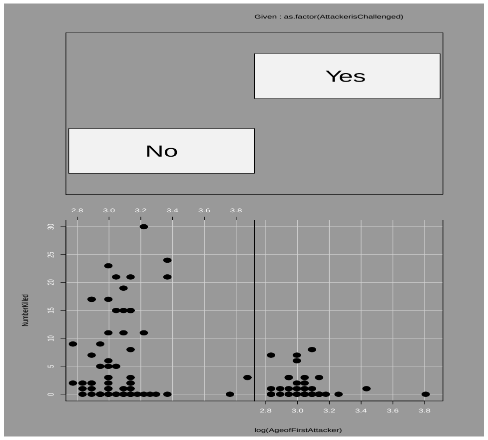
Target is Military



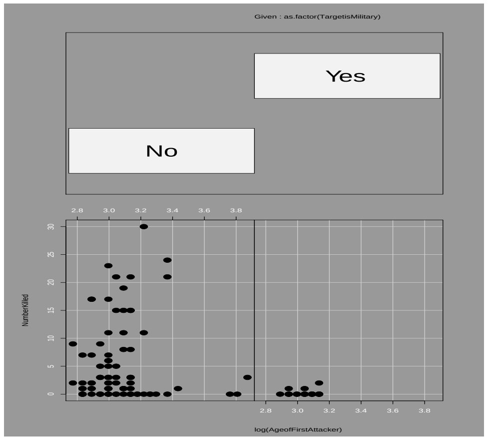
- Much higher level of fatalities for non-military attacks
- These terrorist groups prefer civilian targets.

Background On Terrorism Data
Attacks in Israel – Confounding

Attacker is Challenged



Target is Military



- There is confounding
- Suicide bombers attacking civilian targets are rarely challenged
- So there is little to distinguish between these two plots.

Background On Terrorism Data
Challenges from the Data

- Data on terrorist attacks have special challenges
  - Coarse measurements; many categorical and qualitative variables.
  - Important variables missing: *intentions* and *strategies* of the terrorists
- Assume: Observed events resemble events that failed or were cancelled
- These difficulties in the data-analytic understanding of terrorism
  - Lead us to a Bayesian nonparametric setup
  - Use a rich error structure with Dirichlet process priors
  - Attempt to capture latent variability

———But First———
Here is the Big Picture

▶ Usual Random Effects Model

$$\mathbf{Y}|\psi \sim N(X\beta + \psi, \sigma^2 I), \quad \psi_i \sim N(0, \tau^2)$$

▷ Subject-specific random effect

▶ Dirichlet Process Random Effects Model

$$\mathbf{Y}|\psi \sim N(X\beta + \psi, \sigma^2 I), \quad \psi_i \sim \mathcal{DP}(m, N(0, \tau^2))$$

▶ Results in

▷ Fewer Assumptions

▷ Better Estimates

▷ Shorter Credible Intervals

A Dirichlet Process Random Effects Model
Estimating the Dirichlet Process Parameters

- A general random effects Dirichlet Process model can be written

$$(Y_1, \ldots, Y_n) \sim f(y_1, \ldots, y_n \mid \theta, \psi_1, \ldots, \psi_n) = \prod_i f(y_i|\theta, \psi_i)$$

  - $\psi_1, \ldots, \psi_n$ iid from $G \sim \mathcal{DP}$
  - $\mathcal{DP}$ is the Dirichlet Process
    - Base measure ϕ_0 and precision parameter m
  - The vector θ contains all model parameters

- Blackwell and MacQueen (1973) proved

$$\psi_i|\psi_1, \ldots, \psi_{i-1} \sim \frac{m}{i-1+m}\,\phi_0(\psi_i) + \frac{1}{i-1+m}\sum_{l=1}^{i-1} \delta(\psi_l = \psi_i)$$

  - Where δ denotes the Dirac delta function.

Some Distributional Structure

- Freedman (1963), Ferguson (1973, 1974) and Antoniak (1974)
  - ▷ Dirichlet process prior for nonparametric G
  - ▷ Random probability measure on the space of all measures.

- Notation
  - ▷ G_0, a base distribution (finite non-null measure)
  - ▷ $m > 0$, a precision parameter (finite and non-negative scalar)
    - ▷ Gives spread of distributions around G_0,
  - ▷ Prior specification $G \sim \mathcal{DP}(m, G_0) \in \mathcal{P}$.

- For *any* finite partition of the parameter space, $\{B_1, \ldots, B_K\}$,

$$(G(B_1), \ldots, G(B_K)) \sim \mathcal{D}\left(mG_0(B_1), \ldots, mG_0(B_K)\right),$$

A Mixed Dirichlet Process Random Effects Model
Likelihood Function

- The likelihood function is integrated over the random effects

$$L(\theta \mid \mathbf{y}) = \int f(y_1, \ldots, y_n \mid \theta, \psi_1, \ldots, \psi_n) \pi(\psi_1, \ldots, \psi_n) \, d\psi_1 \cdots d\psi_n$$

- From Lo (1984 Annals) Lemma 2 and Liu (1996 Annals)

$$L(\theta \mid \mathbf{y}) = \frac{\Gamma(m)}{\Gamma(m+n)} \sum_{k=1}^{n} m^k \left[\sum_{C:|C|=k} \prod_{j=1}^{k} \Gamma(n_j) \int f(\mathbf{y}_{(j)} \mid \theta, \ \psi_j) \phi_0(\psi_j) \, d\psi_j \right],$$

  - The partition C defines the subclusters
  - $\mathbf{y}_{(j)}$ is the vector of y_is in subcluster j
  - ψ_j is the common parameter for that subcluster

How Is This Nonparametric?

- These models stipulate uncertainty at the level of distribution functions
  - ▷ Allows for infinite dimensional alternatives
  - ▷ Thus a nonparametric approach
- If $\{f(y|\boldsymbol{\psi}){:}\boldsymbol{\psi} \in (\boldsymbol{\Psi} \subset \Re^d)\}$ is a parametric family of distributions
  - ▷ Construct the family of distributions $\mathcal{F} = \{F_G : G \in \mathcal{P}\}$:

$$f(y|G) = \int f(y|\boldsymbol{\psi})dG(\boldsymbol{\psi}).$$

- Now $\mathcal{F}$ becomes a nonparametric family of mixtures.
- G remains random because it comes from a definable measure
  - ▷ Dirichlet process

Logistic Regression with Random Effects
Setup

- We begin with the model

$$Y_i \sim \text{ Bernoulli}(p(\mathbf{X}_i)), \quad i = 1, \ldots, n$$

where

  - $y_i = \begin{cases} 1 & \text{if the attack is a suicide attack} \\ 0 & \text{if the attack is not a suicide attack} \end{cases}$
  - $p(\mathbf{X}_i) = \mathrm{E}(Y_i|\mathbf{X}_i)$ is the probability of a success
  - $\mathbf{X}_i$ = covariates associated with the i^{th} observation

- Extra variation is modeled with a random effect

$$\text{logit}(p(\mathbf{X}_i)) = \frac{\log(p(\mathbf{X}_i))}{1 - \log(p(\mathbf{X}_i)} = \mathbf{X}_i\boldsymbol{\beta} + \phi_i,$$

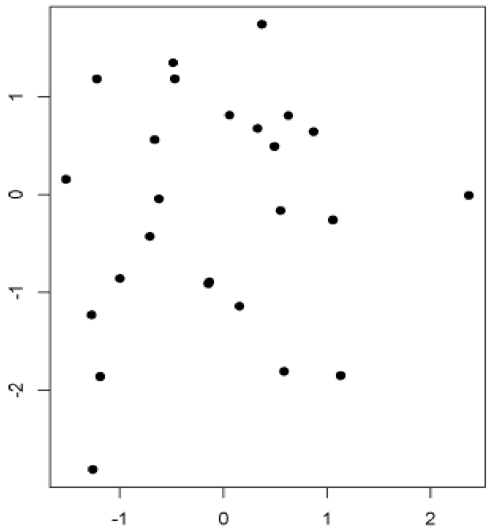
where ϕ_i is a random variable to model extra unexplained variation.

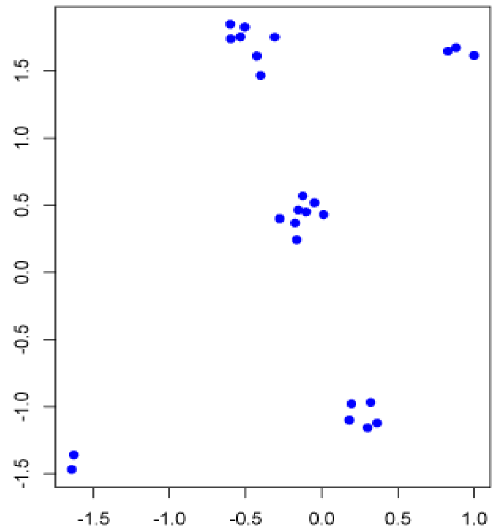
Logistic Regression with Random Effects
Choices for Random Effect Models

- The typical random effect model

$$\mathrm{logit}(p(\mathbf{X}_i)) = \mathbf{X}_i\boldsymbol{\beta} + \phi_i,$$

  - Will often model ϕ_i with a normal distribution
  - We use the alternative ψ_i from a Dirichlet process





- Notice the clustering
- Models extra variability

Logistic Regression with Random Effects
The Full Hierarchical Model

- Observe $Y_i = 0$ or 1 depending on whether the attack was a suicide attack

$$Y_i \sim \text{Bernoulli}(p(\mathbf{X}_i)), \quad i = 1, \ldots, n$$

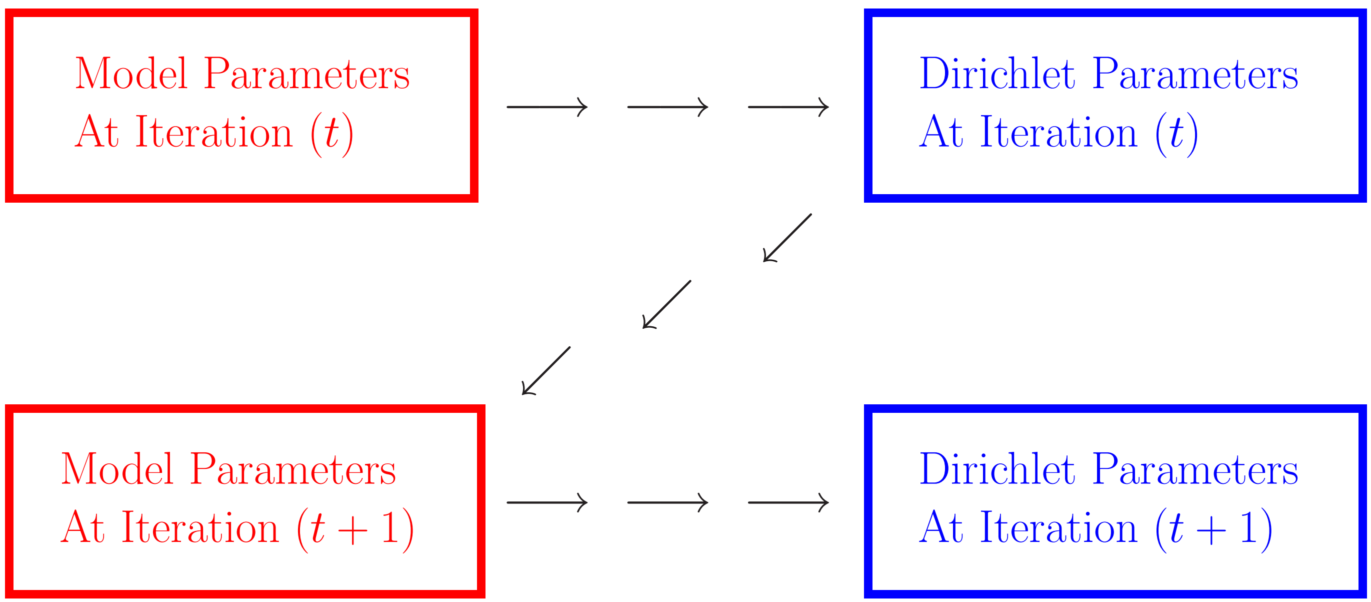
$$\text{logit}(p(\mathbf{X}_i)) = \mathbf{X}_i\boldsymbol{\beta} + \psi_i,$$

- $\boldsymbol{\beta} \sim N\left(\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \sigma^2 I\right)$
  - ▷ $\mu \propto 1$ a flat prior
  - ▷ σ^2 is fixed
- Model Parameters

- $\psi_i \sim G, \quad G \sim \mathcal{DP}(mG_0),$
  - ▷ $G_0 = \text{Normal}(0, \tau^2)$
  - ▷ $\tau^2 \sim$ Inverted Gamma
  - ▷ $m \sim$ Gamma
- Dirichlet Parameters

Fitting the Model
Markov Chain Monte Carlo

- Use a Gibbs Sampler, a Markov Chain Monte Carlo Algorithm.
  - Estimates the posterior distribution of the parameters
  - Gives point estimates and confidence intervals
- Iterates between Model Parameters and Dirichlet Parameters.



Fitting the Logistic Parameters
Mixture Representation

- Logistic is a Mixture of Normals

  - Kolmogorov-Smirnov density:

$$f_{KS}(x) = 8 \sum_{\alpha=1}^{\infty} (-1)^{\alpha+1} \alpha^2 x e^{-2\alpha^2 x^2} \quad x \geq 0$$

  - Mixture of normals is logistic (Andrews and Mallows 1974)

$$\int_0^{\infty} \frac{1}{2x\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{y}{2x}\right)^2\right\} f_{KS}(x)\, dx = \frac{e^{-y}}{(1+e^{-y})^2}$$

- Easy to simulate (Devroye's (1986) Accept-Reject Algorithm)
- Outperforms Slice Sampler

Fitting the Dirichlet Parameters
Matrix Representation of Partitions

- $\boldsymbol{\psi} \sim \mathcal{DP}$
  - $\boldsymbol{\psi} = \mathbf{A}\boldsymbol{\eta}$, $\eta \sim N_k(0, \sigma^2 I)$
- $\mathbf{A}_{n\times k}$ random with
  - Rows: a_i is a $1 \times k$ vector of all zeros except for a 1 in its subcluster
  - Columns: Column sums are the number of observations in the groups
- To Generate $\mathbf{A}$

$$\begin{aligned} \mathbf{q}_{n\times 1} &\sim \text{Dirichlet Distribution} \\ a_i &\sim \text{Multinomial}, \quad i = 1, \dots n \end{aligned} \qquad \mathbf{A} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$$

- Eliminate columns with all zeros (Kyung *et al.* 2010)

Analysis of the Terrorism Data
Background

- The data come from the Global Terrorism Database II
  - Events in the Middle East and Northern Africa from 1998 to 2004
- 1998: 273 attacks worldwide, record high of 741 killed, 5952 injured.
  - Incredibly destructive simultaneous bombings of the U.S. Embassies in Nairobi, Kenya (291 killed, roughly 5000 injured), and Dar es Salaam, Tanzania (10 killed, 77 injured) in August.
- Categorization of Attack Types

	Not Bomb	**Bomb**
Not Suicide	720	661
Suicide	5	224

- Outcome variable: Suicide attack/Not. ← Case-Control
- Suicide attacks pose a substantially higher challenge for governments
  - The assailant has great control over placement and timing
  - Does not need to plan his or her escape (Pape 2006).

Analysis of the Terrorism Data

Some Covariates Used in the Analysis

`MULT.INCIDENT`	Indicates if the attack is part of a coordinated multi-site event
`SUCCESSFUL`	The perceived success rated by the party attacked
`WEAPON.TYPE`	Type of Weapon: Bomb or Other Weapon

Analysis of the Terrorism Data

Other Covariates Used in the Analysis

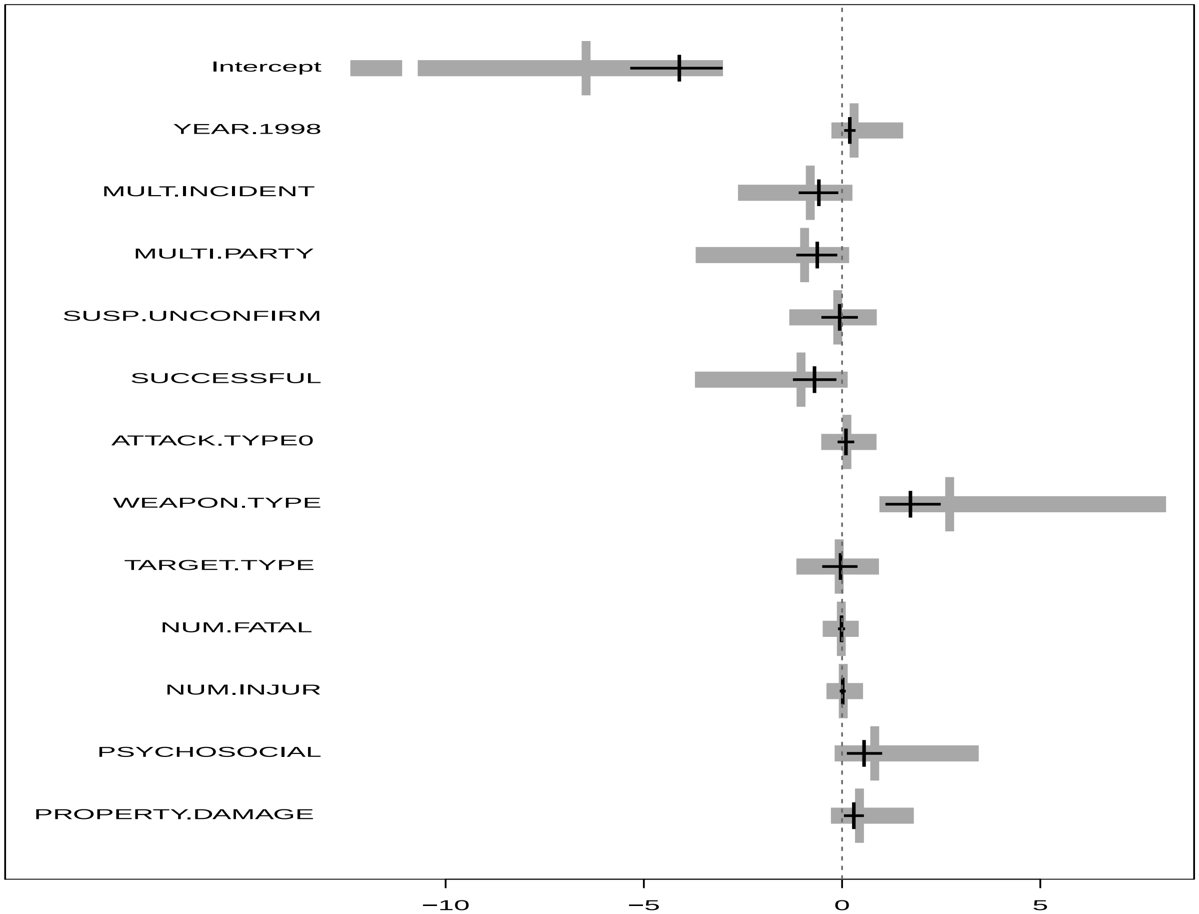
`NUM.INJUR`	Extent of human damage from the terrorist attack.
`PROPERTY.DAMAGE`	Amount of property damage.
`PSYCHOSOCIAL`	The negative psychological/social impact; ascending levels: none, minor, moderate, and major.

But First, We Are Statisticians After All
Model Results, Suicide Attacks

	Standard Bayes Model				GLMDM Logit			
Coefficient	COEF	SE	95% HPD		COEF	SE	95% HPD	
Intercept	-6.457	4.232	-21.605	-3.407	-4.105	0.559	-5.276	-3.079
YEAR - 1998	0.303	0.228	0.135	1.137	0.195	0.039	0.121	0.273
MULT.INCIDENT	-0.802	0.488	-2.222	-0.142	-0.585	0.221	-1.028	-0.162
MULTI.PARTY	-0.945	0.690	-3.289	-0.225	-0.626	0.229	-1.088	-0.189
SUSP.UNCONFIRM	-0.109	0.344	-0.928	0.472	-0.061	0.198	-0.455	0.331
SUCCESSFUL	-1.035	0.705	-3.308	-0.262	-0.695	0.245	-1.172	-0.210
ATTACK.TYPE	0.122	0.135	-0.122	0.466	0.098	0.073	-0.046	0.240
WEAPON.TYPE	2.714	1.673	1.346	7.769	1.725	0.320	1.162	2.422
TARGET.TYPE	-0.073	0.330	-0.749	0.527	-0.038	0.185	-0.434	0.323
NUM.FATAL	-0.019	0.025	-0.085	0.017	-0.013	0.012	-0.036	0.009
NUM.INJUR	0.030	0.030	0.010	0.126	0.017	0.004	0.008	0.025
PROPERTY.DAMAGE	0.439	0.305	0.122	1.406	0.297	0.094	0.114	0.483
PSYCHOSOCIAL	0.824	0.633	0.216	3.044	0.555	0.192	0.188	0.944

- Standard errors are smaller with $\mathcal{DP}$ random effects

Model Results, Suicide Attacks
Grey=Standard, Black=$\mathcal{DP}$



- And the credible intervals tend to be shorter

Analysis of the Terrorism Data
Estimates and Confidence Intervals

Coefficient	Coefficient	Std. Error	95% CI	
MULT.INCIDENT	-0.585	0.221	-1.028	-0.162
SUCCESSFUL	-0.695	0.245	-1.172	-0.210
WEAPON.TYPE	1.725	0.320	1.162	2.422
NUM.INJUR	0.017	0.004	0.008	0.025
PROPERTY.DAMAGE	0.297	0.094	0.114	0.483
PSYCHOSOCIAL	0.555	0.192	0.188	0.944

- Significant Coefficients

Analysis of the Terrorism Data
Results

`MULT.INCIDENT` -0.585^*	Multiple coordinated incidents are less associated with suicide attacks (9/11/2001 an exception)

▶ Planners of simultaneous terrorist events find it difficult to arrange multiple suicidal terrorists.

`SUCCESSFUL` -0.695	Successful attacks are less likely to be from suicides

▶ With suicide attacks, variables such as fervent nationalism and religious extremism, experience, age, intelligence, are important

`WEAPON.TYPE` 1.725	Bomb attacks are more likely to be from suicide terrorists.

Analysis of the Terrorism Data

Results – Continued

`NUM.INJUR` 0.017	More injuries at the event site suggest a greater probability of a suicide attack.
`PROPERTY.DAMAGE` 0.297	Increased property damage is positively associated with a suicide attack.

- This shows the terrorists preference for civilian targets, which will have more damage than better protected military targets.

`PSYCHOSOCIAL` 0.555	A goal of suicide attacks are consequences such as the psychological/social effect.

- A fundamental goal of terrorism is to reduce the people's confidence in the ability of their government to defend them

Conclusions
What Did We Learn From the Model?

- Multiple groups working together do not typically use suicide attackers
- They work in a more military manner with standard weapons

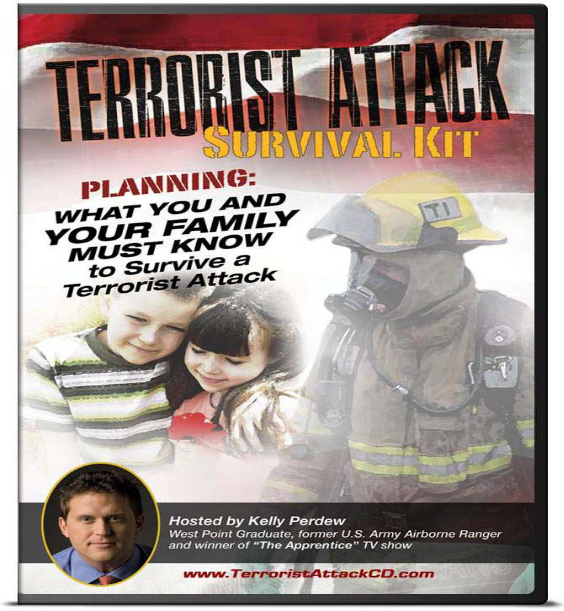
- Increased property damage from suicide attacks.
- Increased human injuries from suicide attacks.
- Suicide attackers prefer civilian targets
- Fewer fatalities from suicide attacks.

Conclusions
From This Model to the Next Step in the Statistical Analysis

- Advantages of the Dirichlet model

- Usual Model
  - Cannot remove enough error variability
  - Over-estimates effects of the covariates

- Dirichlet Model
  - Removes addional error variability
  - Does not over-estimate covariate effects



- We need more explanatory power
  - More covariates
  - More government data
  - Meta-analysis

- These findings may help governments reduce effectiveness of terrorist events.

Conclusions
What Actions are Suggested from the Data Analysis?

- Information on
  - Target/Weapon preferences
  - Multiple/Single attacks

Help focus intelligence gathering

- Plotters of suicide attacks want
  - negative psychological/social impact

- Better education of the population
- Increase availability of counseling

Conclusions

What Actions are Can We Hope For?

- A challenge to the terrorist
  - Reduces success

- Increase Police/Military Presence
- Increase Population Awareness

- We hope for more stories like this

Thank You for Your Attention

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Findings So Far for Dirichlet Process Random Effects in GLMs

- Gill and Casella(2009). "Nonparametric Priors For Ordinal Bayesian Social Science Models: Specification and Estimation." *JASA*, 104, 453-464
  DPP on RE can uncover latent clustering.

- Kyung *et al.*(2009) "Characterizing the Variance Improvement in Linear Dirichlet Random Effects Models." *Stat. Prob. Letters*, 79, 2343-2350
  DPP on RE can produce lower SE for regression parameters on average.

- Kyung, Gill and Casella(2010) "Estimation in Dirichlet Random Effects Models." *Annals of Statistics*, 38, 979-1009
  Estimation of the precision parameter; improved Gibbs sampler.

- Kyung *et al.* (2011) "Sampling Schemes for Generalized Linear Dirichlet Process Random Effects Models." *Stat. Methods & Applications, to appear*.
  Slice sampling worse than KS mixture representation or MH algorithm.

- Kyung *et al.* (2011) "New Findings from Terrorism Data: Dirichlet Process Random Effects Models for Latent Groups." *JRSSC, to appear*.
  Logistic model, uncovering latent information with difficult data.