1. Indicate whether each of the following is true (T) or false (F).

(a) **T** In 2×2 tables, statistical independence is equivalent to a population odds ratio value of $\theta = 1.0$.

(b) **F** A British study reported in the *New York Times* (Dec. 3, 1998) stated that of smokers who get lung cancer, “women were 1.7 times more vulnerable than men to get small-cell lung cancer.” The number 1.7 is a sample odds ratio.

(c) **T** Using data from the Harvard Physician’s Health Study, we find a 95% confidence interval for the relative risk relating having a heart attack to drug (placebo, aspirin) to be $(1.4, 2.3)$. If we had formed the table with aspirin in the first row (instead of placebo), then the 95% confidence interval would have been $(1/2.3, 1/1.4) = (.4, .7)$.

(d) **T** Pearson’s chi-squared test of independence treats both the rows and the columns of the contingency table as nominal scale; thus, if either or both variables are ordinal, the test ignores that information.

(e) **F** For testing independence with random samples, Pearson’s $X^2$ statistic and the likelihood-ratio $G^2$ statistic both have chi-squared distributions for any sample size, as long as the sample was randomly selected.

(f) **T** Fisher’s exact test is a test of the null hypothesis of independence for 2×2 contingency tables that fixes the row and column totals and uses a hypergeometric distribution for the count in the first cell. For a one-sided alternative of a positive association (i.e., odds ratio $> 1$), the P-value is the sum of the probabilities of all those tables that have count in the first cell at least as large as observed, for the given marginal totals.

(g) **F** The difference of proportions, relative risk, and odds ratio are valid measures for summarizing 2×2 tables for either prospective or retrospective (e.g., case-control) studies.

(h) **F** In a 5×2 contingency table that compares 5 groups on a binary response variable, the $G^2$ chi-squared statistic with $df = 4$ for testing independence can be exactly partitioned into 4 separate independent chi-squared statistics that each have $df = 1$ by comparing row 1 to row 5, row 2 to row 5, row 3 to row 5, and row 4 to row 5.

(i) **T** An ordinary regression model that treats the response $Y$ as having a normal distribution is a special case of a generalized linear model, with normal (a.k.a. Gaussian) random component and identity link function.

(j) **F** One question in a recent General Social Survey asked subjects how many times they had had sexual intercourse in the previous month. The sample means were 5.9 for the male respondents and 4.3 for the female respondents; the sample variances were 54.8 and 34.4. The modal response for each gender was 0. Since the response variable is a count, the best way to model this count with gender as an indicator explanatory variable would be to use a Poisson generalized linear model.
2. Each of 100 multiple-choice questions on an exam has five possible answers but one correct response. For each question, a student randomly selects one response as the answer.

   (a) Specify the probability distribution of the student’s number of correct answers on the exam, identifying the parameter(s) for that distribution.

   \[
   \text{Binomial with } n = 100 \text{ and } \pi = 1/5 = 0.20
   \]

   (b) Would it be surprising if the student made at least 50 correct responses? Explain your reasoning.

   Yes. Let \( Y = \text{number of correct answers} \).

   \[
   \mu = n\pi = (100)\left(\frac{1}{5}\right) = 20 \\
   \sigma = \sqrt{n\pi(1-\pi)} = \sqrt{(100)\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)} = \sqrt{16} = 4 \\
   z = \frac{50 - 20}{4} = \frac{30}{4} = 7.5
   \]

   Fifty is 7.5 standard deviations above the mean. It would be very rare to observe a result as large as this if the answers were chosen at random.

3. Consider the following data from a women’s health study (MI is myocardial infarction, i.e., heart attack).

   \[
   \begin{array}{c|cc}
   & \text{MI} & \\
   \text{Oral Contraceptives} & \text{Yes} & \text{No} \\
   \hline
   \text{Used} & 23 & 34 \\
   \text{Never Used} & 35 & 132
   \end{array}
   \]

   (a) Construct a 95% confidence interval for the population odds ratio.

   \[
   \log \hat{\theta} = \log\left(\frac{(23)(132)}{(34)(35)}\right) \\
   \text{SE} = \sqrt{\frac{1}{23} + \frac{1}{132} + \frac{1}{34} + \frac{1}{35}}
   \]

   95% CI for \( \log \theta \): \( \log \hat{\theta} \pm 1.96 \text{SE} \)

   95% CI for \( \theta \): \( \left( e^{\log \hat{\theta} - 1.96 \text{SE}}, e^{\log \hat{\theta} + 1.96 \text{SE}} \right) \)

   (b) Suppose that the answer to part (a) is (1.3, 4.9). Does it seem plausible that the variables are independent? Explain.

   No. Independence implies a population odds-ratio of \( \theta = 1 \), and 1 is not in the CI.

4. For adults who sailed on the Titanic on its fateful voyage, the odds ratio between gender (female, male) and survival (yes, no) was 11.4.

   (a) What is wrong with the interpretation, “The probability of survival for females was 11.4 times that for males.”
The odds ratio is the ratio of the odds of survival for women to that for men. Thus, the odds of survival for females was 11.4 times the odds of survival for males. However, odds are not probabilities, so the same statement cannot be made about the probability of survival.

(b) When would the quoted interpretation be approximately correct? Why?

When the two probabilities are both very small. When both probabilities are small, the odds ratio and the ratio of the probabilities of survival (a.k.a., the relative risk) are approximately equal.

(c) The odds of survival for females equaled 2.9. For each gender, find the proportion who survived.

\[
\text{odds}_F = 2.9 \implies \frac{\text{odds}_F}{1 + \text{odds}_F} = \frac{2.9}{1 + 2.9} = \frac{2.9}{3.9} = \pi_F
\]

\[
\frac{\text{odds}_F}{\text{odds}_M} = 11.4 \implies \text{odds}_M = \frac{\text{odds}_F}{11.4} = \frac{2.9}{11.4}
\]

\[
\implies \frac{\text{odds}_M}{1 + \text{odds}_M} = \frac{\frac{2.9}{11.4}}{1 + \frac{2.9}{11.4}} \times \frac{11.4}{11.4} = \frac{2.9}{11.4 + 2.9} = \frac{2.9}{14.3} = \pi_M
\]

5. Explain what is meant by overdispersion, and explain how it can occur for Poisson generalized linear models for count data.

Overdispersion occurs when the variance of the response is greater than predicted by the model. For a Poisson distribution, the mean and the variance are equal, so in a Poisson generalized linear model, overdispersion occurs when the variance of the response is greater than the mean. This may be caused by subject heterogeneity or other sources of variation unaccounted for in the model.

6. Explain two ways in which the generalized linear model extends the ordinary regression model that is commonly used for quantitative response variables.

In a GLM:

1. the distribution of the response can be something other than a normal distribution;
2. the mean \( \mu \) of the response satisfies \( g(\mu) = \alpha + \beta_1 x_1 + \cdots + \beta_k x_k \) for a link function \( g \).

In an ordinary regression model, responses are assumed to be normally distributed and \( g \) is the identity function, \( g(\mu) = \mu \).
7. In a recent General Social Survey, gender was cross-classified with party identification. The output below shows some results.

(a) Explain what the numbers in the “expected” table represent. Show how to obtain 261.42.

The estimated expected frequencies are the estimated means for each cell under the assumption that gender and party are independent. These are obtained by dividing the product of the row and column totals for each cell by the table total; thus,

\[
261.42 = \frac{577 \times 444}{980}.
\]

(b) Explain how to interpret the p-value given for the Chi-square statistic.

The p-value represents the probability of obtaining a chi-squared statistic as large or larger than the observed value (7.01), assuming that gender and party affiliation are independent. The p-value obtained here, 0.03, is fairly small (< 0.05) and thus provides fairly strong evidence that gender and party affiliation are dependent.

(c) Explain how to interpret the output of the last command (myadjresids). Which counts were significantly higher than one would expect if party identification were independent of gender?

An adjusted (or standardized) residual greater than 2 (less than -2) indicates that the observed frequency in that cell is greater than (less than) would be expected under independence. Thus the numbers of female Democrats and male Republicans were both higher than we would expect if gender and party affiliation were independent.

> gp

<table>
<thead>
<tr>
<th>party</th>
<th>gender</th>
<th>dem</th>
<th>indep</th>
<th>rep</th>
</tr>
</thead>
<tbody>
<tr>
<td>female</td>
<td>279</td>
<td>73</td>
<td>225</td>
<td></td>
</tr>
<tr>
<td>male</td>
<td>165</td>
<td>47</td>
<td>191</td>
<td></td>
</tr>
</tbody>
</table>

> addmargins(gp)

<table>
<thead>
<tr>
<th>party</th>
<th>gender</th>
<th>dem</th>
<th>indep</th>
<th>rep</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>female</td>
<td>279</td>
<td>73</td>
<td>225</td>
<td>577</td>
<td></td>
</tr>
<tr>
<td>male</td>
<td>165</td>
<td>47</td>
<td>191</td>
<td>403</td>
<td></td>
</tr>
<tr>
<td>Sum</td>
<td>444</td>
<td>120</td>
<td>416</td>
<td>980</td>
<td></td>
</tr>
</tbody>
</table>

> gp.chisq <- chisq.test(gp)

> gp.chisq

Pearson’s Chi-squared test

data:  gp
X-squared = 7.0095, df = 2, p-value = 0.03005
8. For the 23 space shuttle flights that occurred before the Challenger mission in 1986, the table below shows the temperature (°F) at the time of the flight and whether at least one of the six primary O-rings suffered thermal distress (1 = yes, 0 = no). The R shows the results of fitting various models to these data.

(a) For the logistic regression model using temperature as a predictor for the probability of thermal distress, calculate the estimated probability of thermal distress at 31°, the temperature at the time of the Challenger flight.

\[
\text{logit}(\hat{\pi}) = 15.043 - 0.232(31) \quad \Rightarrow \quad \hat{\pi} = \frac{\exp(15.043 - 0.232(31))}{1 + \exp(15.043 - 0.232(31))}
\]

(b) At the temperature at which the estimated probability equals 0.5, give a linear approximation for the change in the estimated probability per degree increase in temperature.

\[
\hat{\beta} \hat{\pi}(1 - \hat{\pi}) = -0.232(\frac{1}{2})(\frac{1}{2}) = \frac{-0.232}{4}
\]

(c) Interpret the estimated effect of temperature on the odds of thermal distress.

The odds of thermal distress are estimated to decrease by a factor of \(e^{-0.232}\) for each 1° increase in temperature.

Equivalently, the odds of thermal distress are estimated to increase by a factor of \(e^{0.232}\) for each 1° decrease in temperature.

(d) Test the hypothesis that temperature has no effect, using the likelihood-ratio test. Interpret results.

\[
LR \text{ stat } = 28.267 - 20.315 = 7.952 \quad df = 22 - 21 = 1 \quad p\text{-value} < 0.005
\]

There is very strong evidence that the probability of thermal distress depends on temperature.

(e) Suppose we treat the (0, 1) response as if it has a normal distribution, and fit a linear model for the probability. Report the prediction equation, and find the estimated probability of thermal distress at 31°. Comment on the suitability of this model.

Prediction eqn: \(\hat{\pi} = 2.90 - 0.0374x\) where \(x = \text{temperature}\)

At \(x = 31\): \(\hat{\pi} = 2.90 - 0.0374(31) \approx 2.9 - 1.2 = 1.7\)
This model is obviously not suitable for these data as it can easily yield an estimated probability greater than 1 (or less than 0). It is also unrealistic to model these binary responses as normal random variables with equal variances.

(f) Suppose you also wanted to include in the model the month during which the launch occurred (January, February, etc.). Show how you could add indicator variables to the model to allow this. Explain how to interpret the coefficients of the indicator variables.

Define eleven dummy variables, say,

\[
c_2 = \begin{cases} 
1, & \text{if Feb} \\
0, & o/w 
\end{cases} \\
c_3 = \begin{cases} 
1, & \text{if Mar} \\
0, & o/w 
\end{cases} \\
\vdots \\
c_{12} = \begin{cases} 
1, & \text{if Dec} \\
0, & o/w 
\end{cases}
\]

Model: \( \logit(\pi) = \alpha + \beta_2 c_2 + \beta_3 c_3 + \cdots + \beta_{12} c_{12} + \beta x \)

In this model, \( e^{\beta_j} \) represents the odds ratio of thermal distress in month \( j \) versus January, assuming the same launch temperature in both months.

(g) Refer to the previous part. Explain how you could further generalize the model to allow interaction between temperature and month of the launch, and explain how you could conduct a test to investigate whether you need the interaction terms.

To allow an interaction between temperature and month of the launch, we expand our model by adding terms to the linear predictor corresponding to the products of each of the dummy variables for month with the variable \( x \) (temperature). Thus, our model becomes

\[
\logit(\pi) = \alpha + \beta_2 c_2 + \beta_3 c_3 + \cdots + \beta_{12} c_{12} + \beta x + \gamma_2 c_2 x + \gamma_3 c_3 x + \cdots + \gamma_{12} c_{12} x
\]

To test whether the interaction terms are needed we would fit this model and conduct a likelihood ratio test comparing it to the model fit without the interaction terms. The LR test statistic can be computed as the difference in deviances from the two model fits. The p-value for the test would be obtained by referring the LR test statistic to a chi-square distribution with \( \text{df} = 11 \) (the difference in the number of parameters to be estimated in the two models).
Space shuttle data

<table>
<thead>
<tr>
<th>Ft</th>
<th>Temp</th>
<th>TD</th>
<th>Ft</th>
<th>Temp</th>
<th>TD</th>
<th>Ft</th>
<th>Temp</th>
<th>TD</th>
<th>Ft</th>
<th>Temp</th>
<th>TD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>66</td>
<td>0</td>
<td>2</td>
<td>70</td>
<td>1</td>
<td>3</td>
<td>69</td>
<td>0</td>
<td>4</td>
<td>68</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>67</td>
<td>0</td>
<td>6</td>
<td>72</td>
<td>0</td>
<td>7</td>
<td>73</td>
<td>0</td>
<td>8</td>
<td>70</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>57</td>
<td>1</td>
<td>10</td>
<td>63</td>
<td>1</td>
<td>11</td>
<td>70</td>
<td>1</td>
<td>12</td>
<td>78</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>67</td>
<td>0</td>
<td>14</td>
<td>53</td>
<td>1</td>
<td>15</td>
<td>67</td>
<td>0</td>
<td>16</td>
<td>75</td>
<td>0</td>
</tr>
<tr>
<td>17</td>
<td>70</td>
<td>0</td>
<td>18</td>
<td>81</td>
<td>0</td>
<td>19</td>
<td>76</td>
<td>0</td>
<td>20</td>
<td>79</td>
<td>0</td>
</tr>
<tr>
<td>21</td>
<td>75</td>
<td>1</td>
<td>22</td>
<td>76</td>
<td>0</td>
<td>23</td>
<td>58</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Ft = flight no., Temp = temperature, TD = thermal distress (1 = yes, 0 = no). Data based on Table 1 in *J. Amer. Statist. Assoc.*, 84: 945-957, (1989), by S. R. Dalal, E. B. Fowlkes, and B. Hoadley.

Call:

```
glm(formula = TD ~ Temp, family = binomial, data = shuttle)
```

Deviance Residuals:

```
  Min 1Q Median 3Q Max
-1.061 -0.761 -0.378 0.452 2.217
```

Coefficients:

```
  Estimate Std. Error z value Pr(>|z|)
(Intercept) 15.043     7.379  2.04   0.041
Temp       -0.232     0.108 -2.14   0.032
```

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 28.267  on 22  degrees of freedom
Residual deviance: 20.315  on 21  degrees of freedom
AIC: 24.32

Number of Fisher Scoring iterations: 5
Call:
glm(formula = TD ~ 1, family = binomial, data = shuttle)

Deviance Residuals:
     Min      1Q  Median      3Q     Max
-0.852  -0.852  -0.852  1.542  1.542

Coefficients:
              Estimate Std. Error z value Pr(>|z|)
(Intercept) -0.8270    0.4531  -1.820   0.068

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 28.267 on 22 degrees of freedom
Residual deviance: 28.267 on 22 degrees of freedom
AIC: 30.27

Number of Fisher Scoring iterations: 4

----------------------------------------------------------------------

Call:
lm(formula = TD ~ Temp, data = shuttle)

Residuals:
     Min      1Q  Median      3Q     Max
-0.4376 -0.3068 -0.0638  0.1745  0.8988

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)   2.9048    0.8421   3.450  0.0024
Temp          -0.0374    0.0120  -3.100  0.0054

Residual standard error: 0.399 on 21 degrees of freedom
Multiple R-squared: 0.314, Adjusted R-squared: 0.282
F-statistic: 9.63 on 1 and 21 DF,  p-value: 0.00538
Formulas

Binomial: \( P(y) = \frac{N!}{y!(N-y)!} \pi^y(1-\pi)^{N-y}, \ y = 0, 1, 2, ..., N, \ \mu = N\pi, \ \sigma = \sqrt{N\pi(1-\pi)} \)

Poisson: \( P(y) = \frac{\mu^y e^{-\mu}}{y!}, \ y = 0, 1, 2, 3, ... \)

Hypergeometric: \( P(n_{11}) = \binom{n}{n_{11}} \binom{n_{2+}-n_{11}}{n_{+1}-n_{11}} \binom{n_{11}+n_{22}}{n}, \ \text{where} \ \binom{a}{b} = \frac{a!}{b!(a-b)!} \)

odds = \( \frac{\pi}{1-\pi}, \ \pi = \frac{\text{odds}}{1+\text{odds}}, \ \text{relative risk} = \frac{\pi_1}{\pi_2} \)

\( \theta = \frac{\pi_1/(1-\pi_1)}{\pi_2/(1-\pi_2)}, \ \hat{\theta} = \frac{n_{11}n_{22}}{n_{12}n_{21}}, \ \text{SE}(\log \hat{\theta}) = \sqrt{\frac{1}{n_{11}} + \frac{1}{n_{12}} + \frac{1}{n_{21}} + \frac{1}{n_{22}}} \)

\( X^2 = \sum_{ij} \left( \frac{n_{ij} - \hat{\mu}_{ij}}{\hat{\mu}_{ij}} \right)^2, \ G^2 = 2 \sum_{ij} n_{ij} \log \left( \frac{n_{ij}}{\hat{\mu}_{ij}} \right), \ \hat{\mu}_{ij} = \frac{n_{i+}n_{+j}}{n}, \ \text{df} = (I-1)(J-1) \)

\( r_{ij} = \frac{n_{ij} - \hat{\mu}_{ij}}{\sqrt{\hat{\mu}_{ij}(1-p_{i+})(1-p_{+j})}} \)

LR statistic = \( -2(L_0 - L_1) = \text{difference in deviances} \)

For \( H_0 : \beta = 0, \ \text{Wald statistic} = z = \frac{\hat{\beta}}{\text{SE}} \)

Simple logistic regression: \( \logit[\pi(x)] = \alpha + \beta x \ \pi(x) = \frac{\exp(\alpha + \beta x)}{1 + \exp(\alpha + \beta x)} \)

\( \hat{\pi} = 0.5 \ \text{at} \ x = -\frac{\hat{\alpha}}{\beta}, \ \text{incremental rate of change} = \hat{\beta}(1 - \hat{\pi}), \ \ e^{\hat{\beta}} = \text{estimated odds ratio} \)

\( \logit(\pi) = \alpha + \beta_1 x_1 + \cdots + \beta_k x_k, \ \pi = \frac{\exp(\alpha + \beta_1 x_1 + \cdots + \beta_k x_k)}{1 + \exp(\alpha + \beta_1 x_1 + \cdots + \beta_k x_k)} \)
### Chi-Squared Distribution Values for Various Right-Tail Probabilities

<table>
<thead>
<tr>
<th>df</th>
<th>0.250</th>
<th>0.100</th>
<th>0.050</th>
<th>0.025</th>
<th>0.010</th>
<th>0.005</th>
<th>0.001</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.32</td>
<td>2.71</td>
<td>3.84</td>
<td>5.02</td>
<td>6.63</td>
<td>7.88</td>
<td>10.83</td>
</tr>
<tr>
<td>2</td>
<td>2.77</td>
<td>4.61</td>
<td>5.99</td>
<td>7.38</td>
<td>9.21</td>
<td>10.60</td>
<td>13.82</td>
</tr>
<tr>
<td>3</td>
<td>4.11</td>
<td>6.25</td>
<td>7.81</td>
<td>9.35</td>
<td>11.34</td>
<td>12.84</td>
<td>16.27</td>
</tr>
<tr>
<td>5</td>
<td>6.63</td>
<td>9.24</td>
<td>11.07</td>
<td>12.83</td>
<td>15.09</td>
<td>16.75</td>
<td>20.52</td>
</tr>
<tr>
<td>6</td>
<td>7.84</td>
<td>10.64</td>
<td>12.59</td>
<td>14.45</td>
<td>16.81</td>
<td>18.55</td>
<td>22.46</td>
</tr>
<tr>
<td>7</td>
<td>9.04</td>
<td>12.02</td>
<td>14.07</td>
<td>16.01</td>
<td>18.48</td>
<td>20.28</td>
<td>24.32</td>
</tr>
<tr>
<td>10</td>
<td>12.55</td>
<td>15.99</td>
<td>18.31</td>
<td>20.48</td>
<td>23.21</td>
<td>25.19</td>
<td>29.59</td>
</tr>
<tr>
<td>11</td>
<td>13.70</td>
<td>17.28</td>
<td>19.68</td>
<td>21.92</td>
<td>24.72</td>
<td>26.76</td>
<td>31.26</td>
</tr>
<tr>
<td>12</td>
<td>14.85</td>
<td>18.55</td>
<td>21.03</td>
<td>23.34</td>
<td>26.22</td>
<td>28.30</td>
<td>32.91</td>
</tr>
<tr>
<td>13</td>
<td>15.98</td>
<td>19.81</td>
<td>22.36</td>
<td>24.74</td>
<td>27.69</td>
<td>29.82</td>
<td>34.53</td>
</tr>
<tr>
<td>15</td>
<td>18.25</td>
<td>22.31</td>
<td>25.00</td>
<td>27.49</td>
<td>30.58</td>
<td>32.80</td>
<td>37.70</td>
</tr>
<tr>
<td>16</td>
<td>19.37</td>
<td>23.54</td>
<td>26.30</td>
<td>28.85</td>
<td>32.00</td>
<td>34.27</td>
<td>39.25</td>
</tr>
<tr>
<td>17</td>
<td>20.49</td>
<td>24.77</td>
<td>27.59</td>
<td>30.19</td>
<td>33.41</td>
<td>35.72</td>
<td>40.79</td>
</tr>
<tr>
<td>18</td>
<td>21.60</td>
<td>25.99</td>
<td>28.87</td>
<td>31.53</td>
<td>34.81</td>
<td>37.16</td>
<td>42.31</td>
</tr>
<tr>
<td>19</td>
<td>22.72</td>
<td>27.20</td>
<td>30.14</td>
<td>32.85</td>
<td>36.19</td>
<td>38.58</td>
<td>43.82</td>
</tr>
<tr>
<td>20</td>
<td>23.83</td>
<td>28.41</td>
<td>31.41</td>
<td>34.17</td>
<td>37.57</td>
<td>40.00</td>
<td>45.31</td>
</tr>
<tr>
<td>25</td>
<td>29.34</td>
<td>34.38</td>
<td>37.65</td>
<td>40.65</td>
<td>44.31</td>
<td>46.93</td>
<td>52.62</td>
</tr>
<tr>
<td>30</td>
<td>34.80</td>
<td>40.26</td>
<td>43.77</td>
<td>46.98</td>
<td>50.89</td>
<td>53.67</td>
<td>59.70</td>
</tr>
<tr>
<td>40</td>
<td>45.62</td>
<td>51.81</td>
<td>55.76</td>
<td>59.34</td>
<td>63.69</td>
<td>66.77</td>
<td>73.40</td>
</tr>
<tr>
<td>50</td>
<td>56.33</td>
<td>63.17</td>
<td>67.50</td>
<td>71.42</td>
<td>76.15</td>
<td>79.49</td>
<td>86.66</td>
</tr>
<tr>
<td>60</td>
<td>66.98</td>
<td>74.40</td>
<td>79.08</td>
<td>83.30</td>
<td>88.38</td>
<td>91.95</td>
<td>99.61</td>
</tr>
<tr>
<td>70</td>
<td>77.58</td>
<td>85.53</td>
<td>90.53</td>
<td>95.02</td>
<td>100.43</td>
<td>104.21</td>
<td>112.32</td>
</tr>
<tr>
<td>80</td>
<td>88.13</td>
<td>96.58</td>
<td>101.88</td>
<td>106.63</td>
<td>112.33</td>
<td>116.32</td>
<td>124.84</td>
</tr>
<tr>
<td>90</td>
<td>98.65</td>
<td>107.57</td>
<td>113.15</td>
<td>118.14</td>
<td>124.12</td>
<td>128.30</td>
<td>137.21</td>
</tr>
<tr>
<td>100</td>
<td>109.14</td>
<td>118.50</td>
<td>124.34</td>
<td>129.56</td>
<td>135.81</td>
<td>140.17</td>
<td>149.45</td>
</tr>
</tbody>
</table>

*Source:* Calculated using R, available for free on the net.