8.56
a. Since $n = 800$ is large enough, by CLT the distribution of $\frac{\hat{p} - \mu}{\sigma_{\hat{p}}}$ is approximately $N(0, 1)$, thus

$$P(-z_{1-\alpha/2} \leq \frac{\hat{p} - p}{\sigma_{\hat{p}}} \leq z_{1-\alpha/2}) \approx 1 - \alpha.$$ 

hence, a $(1 - \alpha)100\%$ CI for $p$ is $[\hat{p} - z_{1-\alpha/2}\sigma_{\hat{p}}, \hat{p} + z_{1-\alpha/2}\sigma_{\hat{p}}]$.

\[ \alpha = .02 \Rightarrow z_{1-\alpha/2} = 2.326, \hat{p} = .45, \sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = \sqrt{.45(1 - .45)/800} = 0.01759; \] 

thus, a 98% CI for $p$ is $[.45 - 2.326 \times 0.01759, .45 + 2.326 \times 0.01759]$, or $[.40909, .49091]$.

b. The value .5 is not contained in the above interval. Thus, there is no evidence that a majority of adults feel that movies are getting better.

8.59
a. Same as above, we get a $(1 - \alpha)100\%$ CI for $p$ is $[\hat{p} - z_{1-\alpha/2}\sigma_{\hat{p}}, \hat{p} + z_{1-\alpha/2}\sigma_{\hat{p}}]$.

\[ \alpha = .01 \Rightarrow z_{1-\alpha/2} = 1.645, \hat{p} = .78, \sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = \sqrt{.78(1 - .78)/1030} = 0.01291; \] 

thus, a 90% CI for $p$ is $[.78 - 1.645 \times 0.01291, .78 + 1.645 \times 0.01291]$, or $[.75876, .80124]$.

b. Since $.75 < .75876$, there is evidence that the true population is greater than 75%.

8.85
See the lecture notes, we get a $(1 - \alpha)100\%$ CI for $\mu_1 - \mu_2$ is

$$[(\bar{x} - \bar{y}) - t_{n_1 + n_2 - 2,1-\alpha/2}S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}, (\bar{x} - \bar{y}) + t_{n_1 + n_2 - 2,1-\alpha/2}S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}].$$

$$n_1 = 16, \bar{x} = 11, S_1 = 6, n_2 = 20, \bar{y} = 12, S_2 = 8 \Rightarrow$$

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} = \frac{(16 - 1)6^2 + (20 - 1)8^2}{16 + 20 - 2} = 51.64706.$$ 

\[ \alpha = .05 \Rightarrow t_{n_1 + n_2 - 2,1-\alpha/2} \approx z_{1-\alpha/2} = 1.96, \] 

thus a 95% CI for $\mu_1 - \mu_2$ is

$$[(11 - 12) - 1.96\sqrt{51.64706(\frac{1}{16} + \frac{1}{20})}, (11 - 12) + 1.96\sqrt{51.64706(\frac{1}{16} + \frac{1}{20})}],$$

or $[-5.72449, 3.72449]$.

8.92
We are using the same method as above, $n_1 = 4, \bar{x} = .22, S_1^2 = .001, n_2 = 5, \bar{y} = .17, S_2^2 = .002 \Rightarrow$

$$S_p^2 = \frac{(4 - 1).001 + (5 - 1).002}{4 + 5 - 2} = \frac{.011}{7}.$$
\( \alpha = .05 \Rightarrow t_{n_1+n_2-2,1-\alpha/2} = t_{7.975} = 2.365 \), thus a 95% CI for \( \mu_1 - \mu_2 \) is
\[
[(.22 - .17) - 2.365 \sqrt{\frac{.011}{7} \left( \frac{1}{4} + \frac{1}{5} \right)}, (.22 - .17) + 2.365 \sqrt{\frac{.011}{7} \left( \frac{1}{4} + \frac{1}{5} \right)}],
\]
or \([-0.1289, 0.11289]\).

9.1
See Exercise 8.8.
\( V(\hat{\theta}_1) = \theta^2, V(\hat{\theta}_2) = \frac{\theta^2}{2}, V(\hat{\theta}_3) = \frac{5\theta^2}{9}, V(\hat{\theta}_5) = \frac{\theta^2}{3}. \) Since all are unbiased estimators of \( \theta \),
\( MSE(\hat{\theta}_1) = \theta^2, MSE(\hat{\theta}_2) = \frac{\theta^2}{2}, MSE(\hat{\theta}_3) = \frac{5\theta^2}{9}, MSE(\hat{\theta}_5) = \frac{\theta^2}{3}. \)

\[
eff(\hat{\theta}_1, \hat{\theta}_5) = \frac{MSE(\hat{\theta}_5)}{MSE(\hat{\theta}_1)} = \frac{\frac{\theta^2}{3}}{\theta^2} = \frac{1}{3}
\]
\[
eff(\hat{\theta}_2, \hat{\theta}_5) = \frac{MSE(\hat{\theta}_5)}{MSE(\hat{\theta}_2)} = \frac{\frac{\theta^2}{3}}{\frac{\theta^2}{2}} = \frac{2}{3}
\]
\[
eff(\hat{\theta}_3, \hat{\theta}_5) = \frac{MSE(\hat{\theta}_5)}{MSE(\hat{\theta}_3)} = \frac{\frac{\theta^2}{3}}{\frac{5\theta^2}{9}} = \frac{3}{5}
\]

9.4
\( f(y_{(1)}) = n[1 - F(y)]^{n-1} f(y), \)
\( f(y_{(n)}) = n[F(y)]^{n-1} f(y). \)

Since \( Y_1 \sim uniform(0, \theta), \)
\[
f(y) = \begin{cases} \frac{1}{\theta}, & 0 < y < \theta \\ 0, & \text{otherwise} \end{cases}
\]
\[
F(y) = \begin{cases} 0, & y \leq 0 \\ \frac{y}{\theta}, & 0 < y < \theta \\ 1, & y \geq \theta \end{cases}
\]
\[
f(y_{(1)}) = \begin{cases} n[1 - \frac{y}{\theta}]^{n-1} \frac{1}{\theta} = \frac{n(\theta-y)^{n-1}}{\theta^n}, & 0 < y < \theta \\ 0, & \text{otherwise} \end{cases}
\]
\[ E(y_{(1)}) = \int_{0}^{\theta} f(y) \, dy \]
\[ = \int_{0}^{\theta} \frac{n(\theta - y)^{n-1}}{\theta^n} \, dy \]
\[ = \int_{0}^{\theta} \frac{1}{\theta^n} [-(\theta - y)^n]' y \, dy \]
\[ = -\frac{y}{\theta^n} (\theta - y)^n \bigg|_{y=0}^{\theta} + \int_{0}^{\theta} \frac{1}{\theta^n} (\theta - y)^n \, dy \]
\[ = 0 - \frac{1}{\theta^n} \frac{1}{n+1} (\theta - y)^{n+1} \bigg|_{y=0}^{\theta} \]
\[ = \frac{\theta}{n+1} \]

\[ E(y_{(1)}^2) = \int_{0}^{\theta} f(y) y^2 \, dy \]
\[ = \int_{0}^{\theta} \frac{n(\theta - y)^{n-1}}{\theta^n} y^2 \, dy \]
\[ = \int_{0}^{\theta} \frac{1}{\theta^n} [-(\theta - y)^n]' y^2 \, dy \]
\[ = -\frac{y^2}{\theta^n} (\theta - y)^n \bigg|_{y=0}^{\theta} + 2 \int_{0}^{\theta} \frac{y}{\theta^n} (\theta - y)^n \, dy \]
\[ = 0 + 2 \int_{0}^{\theta} \frac{1}{\theta^n} \frac{1}{n+1} \frac{1}{n+1} (\theta - y)^{n+1} \bigg|_{y=0}^{\theta} \]
\[ = -2 \frac{1}{\theta^n} \frac{1}{n+1} \frac{1}{n+1} (\theta - y)^{n+2} \bigg|_{y=0}^{\theta} + 2 \int_{0}^{\theta} \frac{1}{\theta^n} \frac{1}{n+1} y \, dy \]
\[ = 0 - 2 \frac{1}{\theta^n} \frac{1}{n+1} (\theta - y)^{n+2} \bigg|_{y=0}^{\theta} + 2 \frac{\theta^2}{(n+2)(n+1)} \]
\[ = 2 \frac{\theta^2}{(n+2)(n+1)} \]

\[ V(Y_{(1)}) = E(Y_{(1)}^2) - [E(Y_{(1)})]^2 = \frac{2\theta^2}{(n+2)(n+1)} - \left( \frac{\theta}{n+1} \right)^2 = \frac{2\theta^2(n+1) - \theta^2(n+2)}{(n+1)^2(n+2)} = \frac{\theta^2n}{(n+1)^2(n+2)} \]

Since \( \hat{\theta}_1 = (n+1)Y_{(1)} \Rightarrow V(\hat{\theta}_1) = (n+1)^2V(Y_{(1)}) = \frac{\theta^2n}{n+2} \).

\[ f(y_{(n)}) = \begin{cases} 
\frac{n[y]^n-1}{\theta^n} = \frac{ny^{n-1}}{\theta^n}, & 0 < y < \theta \\
0, & \text{otherwise}
\end{cases} \]
\[
E(y_n) = \int_0^\theta \frac{ny^{n-1}y}{\theta^n} dy = \int_0^\theta ny^2 \frac{1}{\theta^n} dy = \frac{ny^{n+1}}{(n+1)\theta^n} \bigg|_{y=0} = n\theta^{n+1} \frac{1}{n+1}
\]

\[
E(y^2_n) = \int_0^\theta \frac{ny^{n-1}y^2}{\theta^n} dy = \int_0^\theta ny^{n+1} \frac{1}{\theta^n} dy = \frac{ny^{n+2}}{(n+2)\theta^n} \bigg|_{y=0} = n\theta^2 \frac{1}{n+2}
\]

\[
V(Y_n) = E(Y^2_n) - [E(Y_n)]^2 = \frac{n\theta^2}{n+2} - \left( \frac{n\theta}{n+1} \right)^2 = \frac{n\theta^2(n+1)^2 - n^2\theta^2(n+2)}{(n+1)^2(n+2)} = \frac{\theta^2 n}{(n+1)^2(n+2)}
\]

Since \( \hat{\theta}_2 = \frac{n+1}{n} Y_n \Rightarrow V(\hat{\theta}_2) = (\frac{n+1}{n})^2 V(Y_n) = (\frac{n+1}{n})^2 \frac{\theta^2 n}{(n+1)^2(n+2)} = \frac{\theta^2}{n(n+2)} \). Since both \( \hat{\theta}_1 \) and \( \hat{\theta}_2 \) are unbiased estimator for \( \theta \), \( MSE(\hat{\theta}_1) = \frac{\theta^2}{n+2} \) and \( MSE(\hat{\theta}_2) = \frac{\theta^2}{n(n+2)} \),

\[
eff(\hat{\theta}_1, \hat{\theta}_2) = \frac{MSE(\hat{\theta}_2)}{MSE(\hat{\theta}_1)} = \frac{\theta^2(n+2)}{n(n+1)} \frac{n}{\theta^2 n} = 1 \frac{1}{n^2}.
\]

9.7

\( Y_1 \sim \exp(\theta) \Rightarrow E(Y_1) = \theta, V(Y_1) = \theta^2 \).

\( \hat{\theta}_2 = \bar{Y}, E(\bar{Y}) = E\left( \frac{\sum_{i=1}^n Y_i}{n} \right) = \frac{\sum_{i=1}^n E(Y_i)}{n} = \frac{nE(Y_1)}{n} = E(Y_1) = \theta \).

Since \( Y_i's \) are i.i.d, \( V(\bar{Y}) = V\left( \frac{\sum_{i=1}^n Y_i}{n} \right) = \frac{\sum_{i=1}^n V(Y_i)}{n^2} = \frac{nV(Y_1)}{n^2} = \frac{\theta^2}{n} \Rightarrow MSE(\hat{\theta}_2) = \frac{\theta^2}{n} \).

\[
eff(\hat{\theta}_1, \hat{\theta}_2) = \frac{MSE(\hat{\theta}_2)}{MSE(\hat{\theta}_1)} = \frac{\theta^2}{n} \frac{1}{\theta^2} = \frac{1}{n}.
\]