a: simple hypothesis  
\[ \Lambda_{1} = \frac{f_{0}(x_{1})}{f_{a}(x_{1})} = \frac{0.2}{0.1} = 2 \]
\[ \Lambda_{2} = \frac{f_{a}(x_{2})}{f_{0}(x_{2})} = \frac{0.3}{0.6} = 0.5 \]
\[ \Lambda_{3} = \frac{f_{a}(x_{3})}{f_{0}(x_{3})} = \frac{2.5}{3} = 0.83 \]
\[ \Lambda_{4} < \Lambda_{2} < \Lambda_{1} < \Lambda_{3} \]

b: simple  
\[ \alpha = 0.2 \]
\[ P(\frac{T(x)}{f_{a}(x)} \leq 0.5) = P(X = x_{4}) = 0.2 \]
\[ \therefore \text{when } X = x_{4} \text{ we reject } H_{0} \]

\[ \alpha = 0.5 \]
\[ P(\frac{T(x)}{f_{a}(x)} \leq 0.75) = P(X = x_{4} \text{ or } X = x_{2}) = 0.3 + 0.2 - 0.5 \]
\[ \therefore \text{when } X = x_{4} \text{ or } X = x_{2} \text{ we reject } H_{0} \]

c: composite hypothesis  
\[ \Lambda = \frac{\{x_{1}, \ldots, x_{n} | \theta_{0}\}}{f_{X}(x_{1}, \ldots, x_{n} | \theta_{0})} = \frac{g(T(x_{1}, \ldots, x_{n}, \theta_{0})) h(x_{1}, \ldots, x_{n})}{g(T(x_{1}, \ldots, x_{n}), \theta)} \]
\[ = \frac{g(T(x_{1}, \ldots, x_{n}, \theta_{0}))}{g(T(x_{1}, \ldots, x_{n}), \theta)} = A(T) \text{ this is a function of } T \]

If the distribution of } T \text{ is known, we can use this distribution to find rejection region: } \{ \Lambda < C \}
\[ \alpha = \beta \implies \{ \Lambda < C \} = P \mid A(T) < C \} \]