1. Problem 6 of Chapter 12

2. Problem 2 of Chapter 13

3. Problem 10 of Chapter 14

4. Problem 11 of Chapter 14 (May assume that \( A \) is symmetric).

5. Let \( A \) be an \( n \times n \) matrix and \( x \) be a \( n \times 1 \) vector. Show that
   
   (a) If \( Ax = 0 \) for all \( x \), then \( A = 0 \).
   
   (b) If \( A \) is symmetric and \( x'Ax = 0 \) for all \( x \), then \( A = 0 \).
   
   (c) If \( A \) is not be symmetric, then \( x'Ax = 0 \) for all \( x \) implies \( A = -A' \).

6. If \( A \) is skew symmetric, i.e. \( A' = -A \), prove that
   
   (a) \( a_{ii} = 0 \) and \( a_{ij} = -a_{ji} \)
   
   (b) \( I + A \) is positive definite.

7. (a) If \( A \) is idempotent and symmetric, prove that it is n.n.d.
   
   (b) When \( X \) and \( Y \) are idempotent, prove that \( XY \) is idempotent given that \( X \) and \( Y \) commute in multiplication, i.e. \( XY = YX \).
   
   (c) Prove that \( I + KK' \) is p.d., for real \( K \).

R Exercises:

1. Using importance sampling estimate

   \[
   \int_0^\infty 2\sin \left( \frac{\pi}{1.5}x \right) \frac{x^{1.65-1}}{p(x)} e^{-\frac{x^2}{2}} dx,
   \]

   where \( p(x) \) is the kernel of a distribution that is somewhat similar to \( \chi^2 \) distribution. Show your steps, the sampling distribution you will use and the reasons for it, etc.

   **Hints:**
   
   - Note \( p(x) \) is missing a normalizing constant.
   
   - Choose a sampling distribution that bounds the variance. Pay special attention to the ratio component of the variance.
   
   - Although \( x \geq 0 \), you may choose a sampling distribution that includes \( x < 0 \) values, but you must “truncate” it (and re-normalize the normalizing constant).