Some Strong and Weak Limit Theorems for Weighted Sums of I.I.D. Banach Space Valued Random Elements with Slowly Varying Weights

Andrew Rosalsky\(^1,\)\(^*\) and Robert L. Taylor\(^2\)

\(^1\)Department of Statistics, University of Florida, Gainesville, FL 32611
\(^2\)Department of Mathematical Sciences, Clemson University, Clemson, SC 29634

**Abstract**

For a sequence of independent and identically distributed random elements \(\{V_n, n \geq 1\}\) in a real separable Banach space \(\mathcal{X}\) and a monotone (increasing or decreasing) positive slowly varying function \(L\), it is shown that the normed weighted sum \(U_n = (\sum_{j=1}^{n} V_j/L(j))/(n/L(n))\) converges almost surely to an element \(v \in \mathcal{X}\) if and only if \(E||V_1|| < \infty\) and \(EV_1 = v\). No geometric conditions on \(\mathcal{X}\) are required for this result. Moreover, when \(\mathcal{X}\) is of Rademacher type \(p\) (\(1 < p \leq 2\)), conditions are provided (which are weaker than \(E||V_1|| < \infty\)) which ensure that \(U_n\) converges in probability to...
some element $v \in \mathcal{X}$. This result can fail without the Rademacher type $p$ hypothesis. Both of these limit theorems are new results even when $\mathcal{X}$ is the real line.

*Key Words:* Real separable Banach space; Rademacher type $p$ Banach space; Independent and identically distributed random elements; Almost sure convergence; Convergence in probability; Slowly varying function; Normed weighted sums