Mean Convergence Theorems with or without Random Indices for Randomly Weighted Sums of Random Elements in Rademacher Type $p$ Banach Spaces

Andrew Rosalsky\textsuperscript{1,*}, M. Sreehari\textsuperscript{2}, and Andrei I. Volodin\textsuperscript{3}

\textsuperscript{1}Department of Statistics, University of Florida, Gainesville, FL 32611

\textsuperscript{2}Department of Statistics, Faculty of Science, Maharaja Sayajirao University of Baroda, Vadodara - 390 002, India

\textsuperscript{3}Department of Mathematics and Statistics, University of Regina, Regina, Saskatchewan S4S 0A2, Canada

Abstract

Some mean convergence theorems are established for randomly weighted sums of the form $\sum_{j=1}^{k_n} A_{nj} V_{nj}$ and $\sum_{j=1}^{T_n} A_{nj} V_{nj}$ where $\{A_{nj}, j \geq 1, n \geq 1\}$ is an array of random variables, $\{V_{nj}, j \geq 1, n \geq 1\}$ is an array of mean 0 random elements in a separable real Rademacher type $p$ ($1 \leq p \leq 2$) Banach space, and $\{k_n, n \geq 1\}$ and $\{T_n, n \geq 1\}$

\textsuperscript{*}Corresponding author. Fax: (352) 392-5175; E-mail: rosalsky@stat.ufl.edu
\{T_n, n \geq 1\} are sequences of positive integers and positive integer-valued random variables, respectively. The results take the form \( \| \sum_{j=1}^{k_n} A_{nj} V_{nj} \| \overset{L_r}{\to} 0 \) or \( \| \sum_{j=1}^{T_n} A_{nj} V_{nj} \| \overset{L_r}{\to} 0 \) where \( 1 \leq r \leq p \). It is assumed that the array \{\( A_{nj} V_{nj}, j \geq 1, n \geq 1 \)\} is comprised of rowwise independent random elements and that for all \( n \geq 1 \), \( A_{nj} \) and \( V_{nj} \) are independent for all \( j \geq 1 \) and \( T_n \) and \{\( A_{nj} V_{nj}, j \geq 1 \)\} are independent. No conditions are imposed on the joint distributions of the random indices \{\( T_n, n \geq 1 \)\}. The sharpness of the results is illustrated by examples.

**Key Words:** Separable real Rademacher type \( p \) Banach space; Array of rowwise independent random elements; Weighted sums; Random weights; Random indices; Mean convergence