A WEAK LAW WITH RANDOM INDICES FOR
RANDOMLY WEIGHTED SUMS OF ROWWISE
INDEPENDENT RANDOM ELEMENTS IN
RADEMACHER TYPE $p$ BANACH SPACES

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ABSTRACT: For randomly weighted and randomly indexed sums of the form
\[ \sum_{j=1}^{T_n} A_{nj} \left( V_{nj} - E(V_{nj}I(||V_{nj}|| \leq c_n)) \right) \]
where \( \{A_{nj}, j \geq 1, n \geq 1\} \) is an array of rowwise independent random variables, \( \{V_{nj}, j \geq 1, n \geq 1\} \) is an array of rowwise independent random elements in a separable real Rademacher type $p$ Banach space, \( \{c_n, n \geq 1\} \) is a sequence of positive constants, and \( \{T_n, n \geq 1\} \) is a sequence of positive integer-valued random variables, we present conditions under which the general weak law of large numbers \( \sum_{j=1}^{T_n} A_{nj} \left( V_{nj} - E(V_{nj}I(||V_{nj}|| \leq c_n)) \right) \overset{P}{\rightarrow} 0 \) holds. It is not assumed that the \( \{V_{nj}, j \geq 1, n \geq 1\} \) have expected values or absolute moments. The sequences \( \{A_{nj}, j \geq 1\} \) and \( \{V_{nj}, j \geq 1\} \) are assumed to be independent for all \( n \geq 1 \). However, no conditions are imposed on the joint distributions of the random indices \( \{T_n, n \geq 1\} \) and no independence conditions are imposed between \( \{T_n, n \geq 1\} \) and \( \{A_{nj}, V_{nj}, j \geq 1, n \geq 1\} \). The sharpness of the weak law is illustrated by examples.

Key words and phrases: Separable real Rademacher type $p$ Banach space; array of rowwise independent random elements; weighted sums; random weights; random indices; weak law of large numbers; convergence in probability.

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