Nonlinear Path Models With Continuous Variables

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Abstract

Linear path models are commonly used to describe the interrelationships among a sequence of continuous variables that are assumed to be causally ordered. Such models consist of a set of recursive linear structural equations, one for each variable in the causal chain. Under the assumption that the errors in these equations are independent with zero expectations, the method of least squares provides consistent, asymptotically normal estimators of model parameters. These estimators are used, along with the well-known Calculus of Coefficients, to estimate the direct effect and all possible indirect effects of any variable in the chain on any subsequent variable. Unfortunately, the Calculus of Coefficients does not hold for nonlinear path models.

In this paper, we derive an analog to the Calculus of Coefficients for nonlinear path models. This methodology, which we call the “Calculus of Effects”, results in a partitioning of the total effect into the sum of an expected direct effect and expectations of all possible indirect effects through intermediate variables in the causal chain.

Techniques for estimation and testing of expected direct and indirect effects are given. Estimates of model coefficients are based on Nonlinear Least Squares methods. Estimation of expected direct and indirect effects will, under most nonlinear model specifications, require Monte Carlo integration techniques. Testing of each expected indirect effect is based on the Intersection-Union Test.

An application of the Calculus of Effects in the field of maternal and child health is presented. Expected direct and indirect effects of mother’s age on child’s mental development, through intermediate variables (i.e., child’s birth weight or length of stay in the hospital at birth), are estimated and tested in an attempt to separately assess the effects of environmental/social and biological/medical factors presumed to be associated with increasing mother’s age.

Key Words: Calculus of Coefficients; Total effect; Direct effect; Indirect effect; Path Analysis; Causal modeling.

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