Optimal sampling designs for processes with a product covariance structure

Bhramar Mukherjee
Department of Statistics
University of Florida
Gainesville FL 32611
USA
email:mukherjee@stat.ufl.edu

February 28, 2002

Abstract

A second-order random process \( Y(t) \), with \( E(Y(t)) \equiv 0 \), is sampled at a finite number of design points \( t_1, t_2, ..., t_n \). On the basis of these observations, one wants to estimate the values of the process at unsampled points using the best linear unbiased estimator (BLUE). The performance of the estimator is measured by a weighted integrated mean squared error. The goal is to find \( t_1, t_2, ..., t_n \), such that this integrated mean squared error (IMSE) is minimized for a fixed \( n \). For processes with a product type covariance structure, i.e., for \( \text{Cov}(Y(s), Y(t)) = u(s)v(t), s < t \), in this paper we obtain a set of necessary and sufficient conditions for a design to be exactly optimal. Explicit calculations of optimal designs for any given \( n \) for Brownian Motion, Brownian Bridge and Ornstein-Uhlenbeck process illustrate the simplicity and usefulness of these conditions. Starting from the set of exact optimality conditions for a fixed \( n \), an asymptotic result yielding the density whose percentile points furnish a set of asymptotically optimal design points (in some suitable sense) is derived. For a more general covariance structure, satisfying natural regularity conditions, some interesting asymptotic results are presented. For processes with no quadratic mean derivative, a much simpler estimator is shown to be asymptotically equivalent to the BLUE. This leads to an intuitively appealing argument in establishing the asymptotic behavior of the BLUE and also in deriving an analytical expression for the asymptotically optimal design density.

Keywords and Phrases: product covariance structure, integrated mean squared error, exact optimal design, Sacks-Ylvisaker regularity conditions, asymptotic optimality.

AMS subject classification: 62K05, 62M09