Abstract

Some mean convergence theorems are established for randomly weighted sums of the form \(\sum_{j=1}^{k_n} A_{nj}(V_{nj} - \theta_{nj})\) and \(\sum_{j=1}^{T_n} A_{nj}(V_{nj} - \theta_{nj})\) where \(\{A_{nj}, j \geq 1, n \geq 1\}\) is an array of random variables, \(\{V_{nj}, j \geq 1, n \geq 1\}\) is an array of random elements in a separable real, martingale type \(p\) \((1 \leq p \leq 2)\) Banach space, \(\{\theta_{nj}, j \geq 1, n \geq 1\}\) is an array of suitable conditional expectations, and \(\{k_n, n \geq 1\}\) and \(\{T_n, n \geq 1\}\) are sequences of positive integers and positive integer-valued random variables, respectively. These results take the form \(\|\sum_{j=1}^{k_n} A_{nj}(V_{nj} - \theta_{nj})\|_{L^r} \to 0\) or \(\|\sum_{j=1}^{T_n} A_{nj}(V_{nj} - \theta_{nj})\|_{L^r} \to 0\) where \(1 \leq r \leq p\). The sharpness of these results is illustrated by examples. Moreover, for \(0 < r \leq 1\), \(L^r\) convergence results of the forms \(\|\sum_{j=1}^{k_n} A_{nj} V_{nj}\|_{L^r} \to 0\) and \(\|\sum_{j=1}^{T_n} A_{nj} V_{nj}\|_{L^r} \to 0\) are presented without the martingale type \(p\) hypothesis imposed on the underlying Banach space.

Key words and phrases: Separable real Banach space, martingale type \(p\) Banach space, array of random elements, weighted sums, random weights, random indices, mean convergence.

AMS 2000 Subject Classification: Primary 60B12, 60F25; Secondary 60B11, 60F05.