On my honor, I have neither given nor receive unauthorized aid on this examination.

Signature______________________________________
1. Let $X_1, X_2, \ldots, X_n$ be independent random variables with the probability density function

$$f(x) = \begin{cases} 2y, & 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Show that

$$\bar{Y} = \sum_{i=1}^{n} Y_i$$

converges in probability toward a constant as $n \to \infty$, and find the constant.

2. Let $X$ have the following probability density function

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>2</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>p(x)</td>
<td>0.3</td>
<td>0.1</td>
<td>0.4</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Find the probability density function of $Y = 2X + 3$
3. Let $X$ be uniformly distributed over $(0, 1)$.

A. Use the **distribution function technique** to find the distribution function of the random variable $Y = e^X$.

B. Find the probability density function of $Y$. 

5. Let \( X \) be a normal random variable with mean \( \mu \) and variance \( \sigma^2 \). Use the moment-generating approach to find the probability density function of \( Y = \frac{X - \mu}{\sigma} \). Be sure to identify the distribution and the parameters.

6. Let \( X_1, X_2, \ldots, X_n \) be independent geometric random variables, each with the common parameter \( p \). Use the moment-generating approach to find the distribution of \( Y = \sum_{i=1}^{n} X_i \). Be sure to identify the distribution and the parameters.
7. Let $X_1$ and $X_2$ be independent, normal random variables each with a mean of 0 and a standard deviation of 1. Let $Y_1 = X_1 + X_2$ and $Y_2 = X_1 - X_2$.

A. Using the **transformation method**, find the joint probability distribution of $Y_1$ and $Y_2$. 
B. Find the marginal distribution of $Y_1$. Be sure to identify the distribution and the parameters.

C. Find the marginal distribution of $Y_2$. Be sure to identify the distribution and the parameters.

D. Are $Y_1$ and $Y_2$ independent? Justify your answer.
8. An anthropologist wishes to estimate the average height of women for a certain race of people. Suppose the population standard deviation is 2 inches.

A. If she randomly samples 100 women, find the probability that the difference between the sample mean and the true population mean will not exceed 0.2 inches?

B. Suppose the anthropologist wants the difference between the sample mean and the population mean to be less than 0.1 inches, with probability 0.95. How many men should she sample to achieve this objective?