

1. Problem 9 of Chapter 4 (p.47).
2. Problem 3 of Chapter 5 (p.53).
3. (i) Suppose matrix $\mathbf{A}_{n \times n}$ satisfied $\mathbf{A}^2 = \mathbf{A}$. Show that $\text{rank}(\mathbf{A}) = \text{trace}(\mathbf{A})$. (Hint: Use Theorem 4.4.9.) (ii) Let \mathbf{X} be a $n \times k$ matrix and $(\mathbf{X}'\mathbf{X})^{-1}$ exist. Show that rank of the matrix $\mathbf{I} - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$ is $n - k$
4. Let $\{\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_k\}$ be a set of vectors. Show that if they are pairwise orthonormal, they form an independent set, but it is not always true if they are only pairwise independent.
5. An R Programming exercise. (Please hand in your R program and outputs)

A casino manager wished to find a rule to determine whether a die is fair. Suppose that you told him that this is a statistical decision and the conclusion could not be 100% correct. In other words, he has to accept risks of making wrong decisions. Moreover, you warned him that if a die is unfair, but close to fairness, it would be very difficult to detect. He agreed with what you said. To him, throwing away good (fair) dice is no big deal because dice are cheap. But if a unfair dice is discovered by a gambler, the casino may lose a lot of money. After some discussion, the risks he can take are:

- (A) Risk of throwing away a good die = $\Pr\{\text{Throw away a good die after testing}\} = 0.50$,
- (B) Risk of accepting a bad unfair die = $\Pr\{\text{Accept a bad die as good after testing}\} = 0.05$.

He then defined a bad unfair die as the one which has probability 0.25 or higher for any one of its faces to appear. He reasoned that any smaller unbalancedness would be difficult to detect by a gambler in one evening. (The least favorable configuration for detection in this case would be the six faces with appearing probabilities 0.25, 0.15, 0.15, 0.15, 0.15, 0.15.)

Construct a decision rule that satisfies the manager's requirements (A) and (B). Your answer should include (1) how many times the die should be tossed, (2) how to use the result to make a decision and (3) if somebody has another method to do this, how do you define and choose the better one?