STA6329 Exercise 1 Due September 2 (Wednesday, Before class)

1. Exercise Problem 3(a) of Chapter 1 (p.11).

2. Exercise Problem 5 of Chapter 1 (p.11).

3. Problem 6 of Chapter 1. (p.11)

4. Let (x, y) be a point in the two-dimensional space. When the axes are rotated an angle θ counter-clockwise, this point has a new coordinate (x', y') with the new axes satisfying

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

a) Derive this formula.
b) Show that if the axes is rotated twice with angles θ_1 and θ_2, then the coordinate transformation is the same by matrix multiplication, or by direct application of the formula with angle $\theta_1 + \theta_2$.
c) What is an intuitive inverse for this matrix? Give an intuitive reason and show it works by multiplication.

The next two problem requires to use R. Please hand in the computer inputs and outputs.

5. (i) Read data econ.txt in the dataset and covert the numbers into a 10×12 matrix $\boldsymbol{A}$ with 12 months in one row. (ii) Exchange column 2 and 3 of $\boldsymbol{A}$ and define it as $\boldsymbol{B}$. (iii) Multiple row 4 of $\boldsymbol{A}$ by -1.2 and add it to row 1 and defined it as $\boldsymbol{W}$. Output the three matrices.

6. Create a 4×4 matrix $\boldsymbol{A}$ which contains consecutive numbers from 1 to 16 (column 1 with 1, 2, 3, 4, etc.). (i) Let $\boldsymbol{B} = \boldsymbol{X}\boldsymbol{A}$, $\boldsymbol{W} = \boldsymbol{Y}\boldsymbol{A}$ where

$$\boldsymbol{X} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{Y} = \begin{bmatrix} 1 & 0 & 0 & -1.2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Output $\boldsymbol{B}$ and $\boldsymbol{W}$. What is the relation between $\boldsymbol{B}$, $\boldsymbol{W}$ and $\boldsymbol{A}$ and operations in the previous problem (Problem 5)?
(ii) What happenS TO $\boldsymbol{A}$ if we let $\boldsymbol{B} = \boldsymbol{A}\boldsymbol{X}$, $\boldsymbol{W} = \boldsymbol{A}\boldsymbol{Y}$? Explain the results using similar descriptions as those in the previous problem.