Homework 2

7.38 (a) By Definition 7.2, t-distribution with 5 degrees of freedom.

(b) By Definition 7.2, t-distribution with 4 degrees of freedom.

(c) $\bar{Y}$ follows a normal distribution with $\mu = 0, \sigma^2 = 1.5$. So, $\sqrt{5}\bar{Y}$ is standard normal and $(\sqrt{5}\bar{Y})^2$ is chi-square with 1 degree of freedom. Therefore, $5\bar{Y}^2 + Y_6^2$ has a chi-square distribution with 2 degrees of freedom (the two random variables are independent). Now, the quotient

$$2(5\bar{Y}^2 + Y_6^2)/U = [(5\bar{Y}^2 + Y_6^2)/2]/[U/4]$$

has an F-distribution with 2 numerator and 4 denominator degrees of freedom.

7.44 Following Ex. 7.43, we now require

$$P(|\bar{Y} - \mu| \leq .4) = P(-.4 \leq \bar{Y} - \mu \leq .4) \simeq P(\frac{-.4\sqrt{n}}{2.5} \leq Z \leq \frac{.4\sqrt{n}}{2.5}) = .95$$

. Thus , it must be true that $\frac{.4\sqrt{n}}{2.5} = 1.96$, or $n = 150.0625$. So, 151 men should be sampled.

7.57 Let Y_i denote the lifetime of the i^{th} lamp, $i = 1, 2, \ldots, 25$, and the mean and standard deviation are given as 50 and 4, respectively. The random variable of interest is $\Sigma_{i=1}^{25} Y_i$, which is the lifetime of the lamp system. So,

$$P(\Sigma_{i=1}^{25} Y_i \geq 1300) = P(\bar{Y} \geq 52) \simeq P(Z \geq \frac{\sqrt{25}(52 - 50)}{4}) = P(Z \geq 2.5) = .0062.$$

7.72 Using the normal approximation, $P(Y \geq 15) \simeq P(Z \geq \frac{14.5-10}{\sqrt{100(.1)(.9)}}) = P(Z \geq 1.5) = .0668.$

Key 1 & 2 referring to *Key questions in statistics.ppt.*