Stature, Hand, and Foot Length among Turkish Males and Females Ages 17-23 Years

Data on n_M = 80 Males and n_F = 75 Females. Variable Names: gender, height, handLen, footLen

- Create a dataframe with height, handLen, footLen for Females (gender = 2) and label it **X.F**
- Obtain the scatterplot matrix among Height, Hand length, and Foot Length among the Females
- Obtain sample mean vector, variance-covariance and correlation matrices with **colMeans**, **cov**, and **cor** functions
- Create a column vector of 1^s of length n_F and label it **vec1.F**
- Create a matrix of 1/n^s of dimension n_F x n_F and label it **Jn.F** and the n_F x n_F identity matrix **I.F**
- Compute the mean vector $\bar{\mathbf{x}}$, matrix of deviations **E**, and variance-covariance matrix **S** directly
- Obtain the diagonal matrix $\mathbf{D}^{-1/2}$ and use it and the variance-covariance matrix to compute the correlation matrix.
- Obtain the eigenvalues and eigenvectors of the variance-covariance matrix in matrix form
- Confirm the Spectral Decomposition of the variance-covariance matrix
- Obtain the Generalized Variance

Matrix Definitions and Formulas:

$$\underset{n\times p}{\mathbf{X}} = \begin{bmatrix} x_{11} & \cdots & x_{1p} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{np} \end{bmatrix} \qquad \underset{n\times 1}{\mathbf{1}} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \qquad \underset{n\times n}{\mathbf{J}} = \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix} \qquad \underset{n\times n}{\mathbf{I}} = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} \qquad \underset{p\times 1}{\bar{\mathbf{x}}} = \frac{1}{n}\mathbf{X}'\mathbf{1} = \begin{bmatrix} \bar{x}_1 \\ \vdots \\ \bar{x}_p \end{bmatrix} \qquad \underset{n\times p}{\bar{\mathbf{X}}} = \frac{1}{n}\mathbf{J}\mathbf{X} = \begin{bmatrix} \bar{x}_1 & \cdots & \bar{x}_p \\ \vdots & \ddots & \vdots \\ \bar{x}_1 & \cdots & \bar{x}_p \end{bmatrix}$$

$$\underset{n\times p}{\mathbf{E}} = \mathbf{X} - \bar{\mathbf{X}} = \begin{bmatrix} x_{11}-\bar{x}_1 & \cdots & x_{1p}-\bar{x}_p \\ \vdots & \ddots & \vdots \\ x_{n1}-\bar{x}_1 & \cdots & x_{np}-\bar{x}_p \end{bmatrix} \qquad \underset{p\times p}{SSCP = \mathbf{E}'\mathbf{E}} = \begin{bmatrix} \sum_{j=1}^{n}\left(x_{j1}-\bar{x}_1\right)^2 & \cdots & \sum_{j=1}^{n}\left[\left(x_{j1}-\bar{x}_1\right)\left(x_{jp}-\bar{x}_p\right)\right] \\ \vdots & \ddots & \vdots \\ \sum_{j=1}^{n}\left[\left(x_{j1}-\bar{x}_1\right)\left(x_{jp}-\bar{x}_p\right)\right] & \cdots & \sum_{j=1}^{n}\left(x_{jp}-\bar{x}_p\right)^2 \end{bmatrix} \qquad \underset{p\times p}{\mathbf{S}_n} = \frac{1}{n}\mathbf{E}'\mathbf{E}$$

$$\underset{p\times p}{\mathbf{S}} = \frac{1}{n-1}\mathbf{E}'\mathbf{E} \qquad \underset{p\times p}{\mathbf{D}^{1/2}} = \begin{bmatrix} \sqrt{s_{11}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sqrt{s_{pp}} \end{bmatrix} \qquad \mathbf{R} = \mathbf{D}^{-1/2}\mathbf{S}\mathbf{D}^{-1/2} \qquad \text{Generalized Variance} \equiv |\mathbf{S}|$$

Some Useful R Commands:

```
## Matrix or Vector Creation (A a 3x3 variance-covariance matrix, say)
A <- matrix(c(3755,407,545, 407,845,54, 545,54,154),byrow=T,ncol=3)

## Matrix Transpose, and for Square Matrices: Inverse, Determinant, and Trace
A.prime <- t(A) ;   A.inv <- solve(A) ;   A.det <- det(A) ;  A.trace <- sum(diag(A))

## Matrix Multiplication (for compatible matrices A and B: col(A)=row(B)) and scalar multiplication:
AB <- A %*% B ;    cA <- c * A

## Unit vector of length n, nxn Identity matrix and nxn J matrix:
One.n <- matrix(rep(1,n), ncol=1) ; I.n <- diag(n) ;   J.n <- matrix(rep(1,n^2), ncol=n)

## Eigenvalues/Eigenvectors of square matrix in Λ and P matrices and Spectral decomposition
Lambda <- diag(eigen(A)$value) ;   P <- eigen(A)$vector ; P %*% Lambda %*% t(P)

## Obtaining D^1/2 from S:
D.12 <- diag(sqrt(diag(S)))
```