# STA 4702/5701 – Spring 2017 - Exam 1 PRINT Name ____________________

Q.1. A random sample of $n$ = 3 movies was taken, and for each movie, the US theater revenues ($X_1$, in millions) and percent positive revenues on Rotten Tomatoes ($X_2$) were observed. Compute the deviation vectors, sample mean vector, variance-covariance matrix, and correlation matrix: $\mathbf{d}_1, \quad \mathbf{d}_2, \quad \bar{\mathbf{x}}, \quad \mathbf{S}_n, \quad \mathbf{R}$

| Movie | X1 | X2 |
|---|---|---|
| 1 | 120 | 50 |
| 2 | 95 | 75 |
| 3 | 145 | 25 |

$$\mathbf{d}_1 = \begin{bmatrix} \\ \\ \\ \end{bmatrix} \qquad \mathbf{d}_2 = \begin{bmatrix} \\ \\ \\ \end{bmatrix} \qquad \bar{\mathbf{x}} = \begin{bmatrix} \\ \\ \\ \end{bmatrix} \qquad \mathbf{S}_n = \begin{bmatrix} \\ \\ \\ \end{bmatrix} \qquad \mathbf{R} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

Q.2. A variance-covariance matrix Σ has the following eigenvalue/eigenvector pairs. Use the spectral decomposition to obtain **Σ** and use that to obtain **ρ**. $\lambda_1 = 31.466 \quad \mathbf{e}_1 = \begin{bmatrix} -0.840 \\ -0.543 \end{bmatrix} \qquad \lambda_2 = 9.534 \quad \mathbf{e}_2 = \begin{bmatrix} 0.543 \\ -0.840 \end{bmatrix}$

$$\boldsymbol{\Sigma} = \begin{bmatrix} & \\ & \end{bmatrix} \qquad \boldsymbol{\rho} = \begin{bmatrix} & \\ & \end{bmatrix}$$

Q.3. Among a population of English Premier League football players, Heights ($x_1$, in inches) and Weights ($x_2$, in pounds) are Normally distributed with the following Mean and Variance-Covariance matrices:

$$\boldsymbol{\mu} = \begin{bmatrix} 71.71 \\ 168.67 \end{bmatrix} \qquad \boldsymbol{\Sigma} = \begin{bmatrix} 8.21 & 34.45 \\ 34.45 & 301.28 \end{bmatrix}$$

p.3.a. Obtain the correlation between height and weight, ρ, and the angle between the deviation vectors, θ:

$$\mathbf{d}_1 = \mathbf{y}_1 - \mu_1 \mathbf{1} = \begin{bmatrix} x_{11} - \mu_1 \\ \vdots \\ x_{N1} - \mu_1 \end{bmatrix} \text{ and } \mathbf{d}_2 = \mathbf{y}_2 - \mu_2 \mathbf{1} = \begin{bmatrix} x_{12} - \mu_2 \\ \vdots \\ x_{N2} - \mu_2 \end{bmatrix}$$

$\rho =$ ____________________ $\theta =$ ____________________

p.3.b. What is the sampling distribution of the sample mean of a random sample of $n$ = 25 players from this population (ignore the finite population correction factor).

$\overline{\mathbf{X}} \sim$ ________________________________________________

p.3.c. What is the conditional distribution of Height ($X_1$), given Weight is 170 pounds ($x_2$=170).

$f(x_1 | x_2) \equiv$ ________________________________________________

Q.4. A random sample of n = 16 regular season NBA games for Shaquille O'Neal was obtained. We would like to test whether his population mean points scored per game is 25 and population mean rebounds per game is 15. The sample mean vector and covariance matrix are given below, where $x_1$ is points and $x_2$ is rebounds.

$$\boldsymbol{\mu}_0 = \begin{bmatrix} 25 \\ 15 \end{bmatrix} \qquad \bar{\mathbf{x}} = \begin{bmatrix} 25.6 \\ 12.5 \end{bmatrix} \qquad \mathbf{S} = \begin{bmatrix} 79.2 & 10.5 \\ 10.5 & 11.3 \end{bmatrix} \qquad \mathbf{S}^{-1} = \begin{bmatrix} 0.0144 & -0.0134 \\ -0.0134 & 0.1009 \end{bmatrix}$$

p.4.a. Test $H_0 : \boldsymbol{\mu} = \boldsymbol{\mu}_0 \quad H_A : \boldsymbol{\mu} \neq \boldsymbol{\mu}_0$ at $\alpha = 0.05$ significance level

Test Statistic: ____________________________ Rejection Region: ______________________ Reject $H_0$? **Yes** / **No**

p.4.b. Obtain simultaneous 95% Confidence Intervals for $\mu_1$ and $\mu_2$.

95% Confidence Interval for $\mu_1$: _____________________ 95% Confidence Interval for $\mu_2$: _____________________