Chapter 7 Solution

7.1.

	z_1	y	$(z_1-\bar{z}_1)^2$	$(z_1-\bar{z})(y-\bar{y})$	$\hat{y}$	$\hat{\varepsilon}$
	10	15	$(10-10)^2=0$	$0(3)=0$	12	3
	5	9	$(5-10)^2=25$	$(-5)(-3)=15$	5.6667	3.3333
	7	3	$(7-10)^2=9$	$(-3)(-9)=27$	8.2000	−5.2
	19	25	$(19-10)^2=81$	$9(13)=117$	23.4000	1.6
	11	7	$(11-10)^2=1$	$1(-5)=-5$	13.2667	−6.2667
	8	13	$(8-10)^2=4$	$(-2)(1)=-2$	9.4667	3.5333
Sum	60	72	120	152	72	0
Mean	10	12				

$\hat{\beta}_1 = \frac{152}{120} = 1.2667$ $\quad \hat{\beta}_0 = 12 - 1.2667(10) = -0.6667$

$\hat{\underset{\sim}{\beta}} = \begin{bmatrix} -0.6667 \\ 1.2667 \end{bmatrix}$ $\quad \hat{\underset{\sim}{\varepsilon}}'\hat{\underset{\sim}{\varepsilon}} = 3^2 + \dots + 3.5333^2 = 101.4667$

7.2.

	z_1	z_2	y	z_1^*	z_2^*	y^*
	10	2	15	0	−0.49	0.17
	5	3	9	−0.46	−0.32	−0.17
	7	3	3	−0.27	−0.32	−0.52
	19	6	25	0.82	0.16	0.76
	11	7	7	0.09	0.32	−0.29
	8	9	13	−0.18	0.65	0.06
Sum	60	30	72	0	0	~0
Mean	10	5	12			
CSS	120	38	294			
Var	24	7.6	58.8			
SD	4.90	2.76	7.67			
$\sqrt{CSS}$	10.95	6.16	17.15			

$Z^{*\prime}Z^* = \begin{bmatrix} 1.00 & .28 \\ .28 & 1.00 \end{bmatrix}$ $\quad Z^{*\prime}Y^* = \begin{bmatrix} 0.80 \\ 0.21 \end{bmatrix}$

$\hat{\beta}^* = \frac{1}{1-.28^2} \begin{bmatrix} 1.00 & -.28 \\ -.28 & 1.00 \end{bmatrix} \begin{bmatrix} .80 \\ .21 \end{bmatrix}$

$= \frac{1}{0.92} \begin{bmatrix} 0.74 \\ -0.01 \end{bmatrix} = \begin{bmatrix} 0.80 \\ -0.01 \end{bmatrix}$

7.2 continued

$$\hat{\beta}_k = \frac{s_y}{s_k}\hat{\beta}_k^{*} \Rightarrow \hat{\beta}_1 = \frac{7.67}{4.90}(0.80) = 1.25$$

$$\hat{\beta}_2 = \frac{7.67}{2.76}(-0.01) = -0.03$$

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1\bar{z}_1 - \hat{\beta}_2\bar{z}_2 = 12 - 1.25(10) - (-.03)(5) = -0.35$$

$$\Rightarrow \hat{Y} = -0.35 + 1.25z_1 - 0.03z_2$$

7.7. $\underset{n\times 1}{Y} = \underset{n\times(q+1)}{Z_1}\underset{(q+1)\times 1}{\beta_{(1)}} + \underset{n\times(r-q)}{Z_2}\underset{(r-q)\times 1}{\beta_{(2)}} + \underset{n\times 1}{\varepsilon}$ $\qquad r(Z) = r+1$

$r(Z_1) = q+1 \qquad r(Z_2) = r-q$

$$\underset{\sim}{Y} = [Z_1 \vdots Z_2]\begin{bmatrix}\beta_{(1)}\\ \cdots \\ \underset{\sim}{\beta_{(2)}}\end{bmatrix} + \underset{\sim}{\varepsilon}$$ Interest in $\underset{\sim}{\beta_{(2)}}$

$$\hat{\beta} = (Z'Z)^{-1}Z'Y$$ Note: $Y = [Z_2 \vdots Z_1]\begin{bmatrix}\beta_{(2)}\\ \beta_{(1)}\end{bmatrix} + \varepsilon$

$$Z'Z = \begin{bmatrix}Z_2'\\ \cdots \\ Z_1'\end{bmatrix}[Z_2 \vdots Z_1] = \begin{bmatrix}Z_2'Z_2 & Z_2'Z_1\\ Z_1'Z_2 & Z_1'Z_1\end{bmatrix}$$

From 4.12 upperleft portion of $(Z'Z)^{-1}$ is:

$$\left(Z_2'Z_2 - Z_2'Z_1(Z_1'Z_1)^{-1}Z_1'Z_2\right)^{-1} = C^{22}$$

$$(C^{22})^{-1/2}(\hat{\beta}_{(2)} - \beta_{(2)})/\sigma^2 \sim N(0, I)$$

Since: $V\{\hat{\beta}_{(2)} - \beta_{(2)}\} = \sigma^2 C^{22} \qquad E\{\hat{\beta}_{(2)} - \beta_{(2)}\} = 0$

$$V\left\{(C^{22})^{-1/2}(\hat{\beta}_{(2)} - \beta_{(2)})\right\} = (C^{22})^{-1/2}\sigma^2 C^{22}\left[(C^{22})^{-1/2}\right]' = \sigma^2 I$$

7.7 continued

$$\Rightarrow \frac{(\hat{\beta}_{(2)}-\beta_{(2)})'(C^{22})^{-1}(\hat{\beta}_{(2)}-\beta_{(2)})}{\sigma^2} \sim \chi^2_{r-q}$$

$$\frac{[n-(r+1)]s^2}{\sigma^2} \sim \chi^2_{n-(r+1)} \qquad s^2 \perp \{\hat{\beta}_{(2)}\}$$

$$\Rightarrow \frac{(\hat{\beta}_{(2)}-\beta_{(2)})'(C^{22})^{-1}(\hat{\beta}_{(2)}-\beta_{(2)})/(r-q)}{[n-(r+1)]s^2/(n-(r+1))} \sim F_{r-q,\, n-(r+1)}$$

$$\Rightarrow \frac{(\hat{\beta}_{(2)}-\beta_{(2)})'(C^{22})^{-1}(\hat{\beta}_{(2)}-\beta_{(2)})}{(r-q)\, s^2} \sim F_{r-q,\, n-(r+1)}$$

$\Rightarrow$ $(1-\alpha)100\%$ Confidence Region for $\beta^{(2)}$:

$$(\hat{\beta}_{(2)}-\beta_{(2)})'\left(Z_2'Z_2 - Z_2'Z_1(Z_1'Z_1)^{-1}Z_1'Z_2\right)(\hat{\beta}_{(2)}-\beta_{(2)}) \le (r-q)s^2 F_{r-q,\, n-(r+1)}(\alpha)$$

7.8 $H = Z(Z'Z)^{-1}Z'$ w/ diagonal elements h_{jj}

a) $HH = Z(Z'Z)^{-1}\underbrace{Z'Z(Z'Z)^{-1}}_{I}Z' = Z(Z'Z)^{-1}IZ' = Z(Z'Z)^{-1}Z' = H$

$H = H'$

b) $HH = H \Rightarrow h_{jj} = \sum_{i=1}^{n} h_{ij}h_{ji} = \sum_i h_{ij}^2 \Rightarrow h_{jj} > h_{jj}^2 \Rightarrow 0 < h_{jj} < 1$

c) $\text{trace}(H) = \text{trace}(Z(Z'Z)^{-1}Z') = tr(Z'Z(Z'Z)^{-1}) = tr(I_{r+1}) = r+1$

ZY

7.8c. $Z'Z = \begin{bmatrix} n & \sum_j z_j \\ \sum_j z_j & \sum_j z_j^2 \end{bmatrix} \Rightarrow (Z'Z)^{-1} = \frac{1}{n\sum_j z_j^2 - (\sum_j z_j)^2} \begin{bmatrix} \sum z_j^2 & -\sum z_j \\ -\sum z_j & n \end{bmatrix}$

$$\Rightarrow (Z'Z)^{-1} = \frac{1}{n\sum_j (z_j - \bar{z})^2} \begin{bmatrix} \sum_j z_j^2 & -\sum_j z_j \\ -\sum_j z_j & n \end{bmatrix} \qquad \sum_j z_j^2 = \sum_j z_j^2 + \frac{1}{n}\left(\sum_j z_j\right)^2 - \frac{1}{n}\left(\sum_j z_j\right)^2$$

$$\Rightarrow (Z'Z)^{-1} = \frac{1}{n\sum_j (z_j - \bar{z})^2} \begin{bmatrix} \sum_j (z_j - \bar{z})^2 + \frac{1}{n}(n\bar{z})^2 & -\sum_j z_j \\ -\sum_j z_j & n \end{bmatrix}$$

$$\Rightarrow (Z'Z)^{-1} = \begin{bmatrix} \frac{1}{n} + \frac{\bar{z}^2}{\sum_j (z_j - \bar{z})^2} & -\frac{\bar{z}}{\sum_j (z_j - \bar{z})^2} \\ -\frac{\bar{z}}{\sum_j (z_j - \bar{z})^2} & \frac{1}{\sum_j (z_j - \bar{z})^2} \end{bmatrix}$$

$$\Rightarrow h_{jj} = \begin{bmatrix} 1 & z_j \end{bmatrix} (Z'Z)^{-1} \begin{bmatrix} 1 \\ z_j \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{n} + \frac{\bar{z}^2}{\sum_j (z_j - \bar{z})^2} - \frac{z_j \bar{z}}{\sum_j (z_j - \bar{z})^2} \; ; \; \frac{-\bar{z}}{\sum_j (z_j - \bar{z})^2} + \frac{z_j}{\sum_j (z_j - \bar{z})^2} \end{bmatrix} \begin{bmatrix} 1 \\ z_j \end{bmatrix}$$

$$= \frac{1}{n} + \frac{\bar{z}^2}{\sum_j (z_j - \bar{z})^2} - \frac{z_j \bar{z}}{\sum_j (z_j - \bar{z})^2} - \frac{\bar{z} z_j}{\sum_j (z_j - \bar{z})^2} + \frac{z_j^2}{\sum_j (z_j - \bar{z})^2}$$

$$= \frac{1}{n} + \frac{(z_j - \bar{z})^2}{\sum_j (z_j - \bar{z})^2}$$

7.9

$$Z = \begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \qquad Y = \begin{bmatrix} 5 & -3 \\ 3 & -1 \\ 4 & -1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$Z'Z = \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix} \qquad (Z'Z)^{-1} = \begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{10} \end{bmatrix}$$

$$Z'Y = \begin{bmatrix} 15 & 0 \\ -9 & 15 \end{bmatrix} \qquad \underset{\sim}{\hat{\beta}} = (Z'Z)^{-1} Z'Y = \begin{bmatrix} 3 & 0 \\ -\frac{9}{10} & \frac{3}{2} \end{bmatrix}$$

$$\hat{Y} = Z\hat{\beta} = \begin{bmatrix} 4.8 & -3 \\ 3.9 & -1.5 \\ 3 & 0 \\ 2.1 & 1.5 \\ 1.2 & 3 \end{bmatrix} \qquad \hat{\varepsilon} = Y - \hat{Y} = \begin{bmatrix} 0.2 & 0 \\ -0.9 & 0.5 \\ 1 & -1 \\ -0.1 & 0.5 \\ -0.2 & 0 \end{bmatrix}$$

$$Y'Y = \begin{bmatrix} 55 & -15 \\ -15 & 24 \end{bmatrix} \qquad \hat{Y}'\hat{Y} = \begin{bmatrix} 53.1 & -13.5 \\ -13.5 & 22.5 \end{bmatrix}$$

$$\hat{\varepsilon}'\hat{\varepsilon} = \begin{bmatrix} 1.9 & -1.5 \\ -1.5 & 1.5 \end{bmatrix} \qquad Y'Y = \hat{Y}'\hat{Y} + \hat{\varepsilon}'\hat{\varepsilon} \checkmark$$

7.10

$$S = \frac{1}{n-(r+1)} \hat{\varepsilon}'\hat{\varepsilon} = \frac{1}{5-(1+1)} \hat{\varepsilon}'\hat{\varepsilon} = \begin{bmatrix} 0.6333 & -0.5 \\ -0.5 & 0.5 \end{bmatrix}$$

a)

$$V\{\underset{\sim}{\hat{\beta}}_{(1)}\} = 0.6333 \begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{10} \end{bmatrix} = \begin{bmatrix} .1267 & 0 \\ 0 & .0633 \end{bmatrix}$$

$$\hat{Y}_{01} = [1 \quad 0.5] \begin{bmatrix} 3 \\ -\frac{9}{10} \end{bmatrix} = 2.55 \qquad V\{\hat{Y}_{01}\} = [1 \quad 0.5] \begin{bmatrix} .1267 & 0 \\ 0 & .0633 \end{bmatrix} \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

7.10 Continued

$$V\{\hat{Y}_{01}\} = [.1267 \quad .0317]\begin{bmatrix}1\\0.5\end{bmatrix} = .1267 + .0159 = .1426$$

$\Rightarrow$ 95% CI for Mean: $2.55 \pm t_3(.025)\sqrt{.1426}$

$$\equiv 2.55 \pm 3.182(.378) \equiv 2.55 \pm 1.20 \equiv (1.35, 3.75)$$

b) 95% Prediction Interval for Individual Y_{01} @ $z_1 = 0.5$

$$2.55 \pm 3.182\sqrt{1 + .1426} \equiv 2.55 \pm 3.182\,(1.069)$$

$$\equiv 2.55 \pm 3.40 \equiv (-0.85, 5.95)$$

c) $$V\{\hat{\beta}_{(2)}\} = 0.5\begin{bmatrix}\frac{1}{5} & 0\\ 0 & \frac{1}{10}\end{bmatrix} = \begin{bmatrix}\frac{1}{10} & 0\\ 0 & \frac{1}{20}\end{bmatrix}$$

$$\hat{Y}_{02} = [1 \quad 0.5]\begin{bmatrix}0\\1.5\end{bmatrix} = 0 + 0.75 = 0.75$$

$$V\{\hat{Y}_{02}\} = [1 \quad 0.5]\begin{bmatrix}0.10 & 0\\ 0 & 0.05\end{bmatrix}\begin{bmatrix}1\\0.5\end{bmatrix}$$

$$= [0.10 \quad 0.025]\begin{bmatrix}1\\0.5\end{bmatrix} = .10 + .0125 = .1125$$

$$\sqrt{1.1125} = 1.055$$

$t_3(.025/2) = 4.177$ Bonferroni Simultaneous 95% PIs for Y_{01}, Y_{02}

Y_{01}: $2.55 \pm 4.177(1.069) \equiv 2.55 \pm 4.47 \equiv (-1.92; 7.02)$

Y_{02}: $0.75 \pm 4.177(1.055) \equiv 0.75 \pm 4.41 \equiv (-3.66, 5.16)$

7.12 $\mu = \begin{bmatrix} \mu_Y \\ \mu_Z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ -2 \end{bmatrix}$ $\Sigma = \begin{bmatrix} \sigma_{YY} & \sigma_{ZY}' \\ \sigma_{ZY} & \sigma_{ZZ} \end{bmatrix} = \begin{bmatrix} 9 & 3 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

a) Best Linear Predictor $\beta_0 + \beta_1 z_1 + \beta_2 z_2$ for Y

$\beta = \Sigma_{ZZ}^{-1} \sigma_{ZY}$ $\quad \Sigma_{ZZ}^{-1} = \frac{1}{2(1)-1(1)} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

$\Rightarrow \beta = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$\beta_0 = \mu_Y - \beta' \mu_Z = 4 - [2 \;\; -1] \begin{bmatrix} 3 \\ -2 \end{bmatrix} = 4 - 8 = -4$

$\Rightarrow -4 + 2z_1 - z_2 \equiv$ Best Linear Predictor of Y

b) MSE of Best Linear Predictor

$E\{(Y - \beta_0 - \beta' z)^2\} = \sigma_{YY} - \sigma_{ZY}' \Sigma_{ZZ}^{-1} \sigma_{ZY}$

$\sigma_{YY} = 9 \qquad \sigma_{ZY}' \Sigma_{ZZ}^{-1} \sigma_{ZY} = [3 \;\; 1] \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = [2 \;\; -1] \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 5$

$\Rightarrow MSE = 9 - 5 = 4$

c) Population Multiple Correlation Coefficient

$\rho_{Y(Z)} = + \sqrt{\frac{\sigma_{ZY}' \Sigma_{ZZ}^{-1} \sigma_{ZY}}{\sigma_{YY}}} = \sqrt{\frac{5}{9}} = .7454$

d) Partial Correlation $\rho_{YZ_1 \cdot Z_2}$

$\Sigma = \begin{bmatrix} 9 & 3 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ $\cancel{\rho_{YZ_1 \cdot Z_2} = \Sigma_{YZ_1 \cdot Z_1}} = \begin{bmatrix} \Sigma_{YY} & \Sigma_{YZ} \\ \Sigma_{ZY} & \Sigma_{ZZ} \end{bmatrix}$

7.8

7.12 d continued ("Y" = YZ_1, "Z" = Z_2)

$$\Sigma_{YY.Z} = \Sigma_{YY} - \Sigma_{YZ}\Sigma_{ZZ}^{-1}\Sigma_{ZY}$$

$$= \begin{bmatrix} 9 & 3 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix}\left(\frac{1}{1}\right)\begin{bmatrix} 1 & 1 \end{bmatrix} = \begin{pmatrix} 9 & 3 \\ 3 & 2 \end{pmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 2 & 1 \end{bmatrix}$$

$$\Rightarrow \rho_{Y,Z_1 \cdot Z_2} = \frac{2}{\sqrt{8}\sqrt{1}} = 0.7071$$

7.13 $\bar{Z} = \begin{bmatrix} \bar{Z}_1 \\ \bar{Z}_2 \\ \bar{Z}_3 \end{bmatrix} = \begin{bmatrix} 527.74 \\ 54.69 \\ 25.13 \end{bmatrix}$ $S = \begin{bmatrix} 5691.34 & & \\ 600.51 & 126.05 & \\ 217.25 & 23.37 & 23.11 \end{bmatrix}$

jointly normal

a) Predicting $\bar{Z}_1$ from $\bar{Z}_2, \bar{Z}_3$

$$\hat{\mu} = \begin{bmatrix} 527.74 \\ \hline 54.69 \\ 25.13 \end{bmatrix} \quad S = \left[\begin{array}{c|cc} 5691.34 & 600.51 & 217.25 \\ \hline 600.51 & 126.05 & 23.37 \\ 217.25 & 23.37 & 23.11 \end{array}\right]$$

$$S_{ZZ}^{-1} = \frac{1}{126.05(23.11) - 23.37^2}\begin{bmatrix} 23.11 & -23.37 \\ -23.37 & 126.05 \end{bmatrix}$$

$$\approx \frac{1}{2366.86}\begin{bmatrix} 23.11 & -23.37 \\ -23.37 & 126.05 \end{bmatrix}$$

$$\hat{\beta} = S_{ZZ}^{-1}S_{ZY} = \frac{1}{2366.86}\begin{bmatrix} 23.11 & -23.37 \\ -23.37 & 126.05 \end{bmatrix}\begin{bmatrix} 600.51 \\ 217.25 \end{bmatrix}$$

$$= \frac{1}{2366.86}\begin{bmatrix} 8800.65 \\ 13350.44 \end{bmatrix} = \begin{bmatrix} 3.72 \\ 5.64 \end{bmatrix}$$ $\hat{\beta}_0 = \bar{Y} - \hat{\beta}'\bar{Z} = 527.74 - (3.72(54.69) + 5.64(25.13))$ →

7.13 continued

$$\Rightarrow \hat{\beta}_0 = 527.74 - (203.45 + 141.73) = 527.74 - 345.18 = 182.56$$

$\Rightarrow$ Prediction of Z_1 from Z_2, Z_3:

$$\hat{\beta}_0 + \hat{\beta}' z = 182.56 + 3.72\, z_2 + 5.64\, z_3$$

b) Multiple correlation coefficient: $R_{z_1(z_2,z_3)}$

$$R_{z_1(z_2,z_3)} = +\sqrt{\frac{s_{ZY}' S_{ZZ}^{-1} s_{ZY}}{s_{YY}}}$$

$$s_{ZY}' S_{ZZ}^{-1} s_{ZY} = \begin{bmatrix} 600.51 & 217.25 \end{bmatrix} \begin{bmatrix} 3.72 \\ 5.64 \end{bmatrix} = 3459.19$$

$$\Rightarrow R_{z_1(z_2,z_3)} = \sqrt{\frac{3459.19}{5691.34}} = \sqrt{.6078} = .7796$$

c) Partial Correlation $R_{z_1 z_2 \cdot z_3}$

$$S_{YY} = \begin{bmatrix} 5691.34 & 600.51 \\ 600.51 & 126.05 \end{bmatrix} \quad S_{YZ} = \begin{bmatrix} 217.25 \\ 23.37 \end{bmatrix} \quad S_{ZZ} = 23.11$$

$$S_{YZ} S_{ZZ}^{-1} S_{ZY} = \begin{bmatrix} 217.25 \\ 23.37 \end{bmatrix} \frac{1}{23.11} \begin{bmatrix} 217.25 & 23.37 \end{bmatrix}$$

$$= \begin{bmatrix} 2042.30 & 219.69 \\ 219.69 & 23.63 \end{bmatrix} \quad S_{YY} - S_{YZ} S_{ZZ}^{-1} S_{ZY} = \begin{bmatrix} 3649.04 & 380.82 \\ 380.82 & 102.42 \end{bmatrix}$$

$$\Rightarrow R_{z_1 z_2 \cdot z_3} = \frac{380.82}{\sqrt{3649.04}\sqrt{102.42}} = \frac{380.82}{611.34} = 0.6229$$

7.14 $Y \equiv$ Rate of Return $\quad Z_1 \equiv$ Attitude to Risk (cons. to risky) $\quad Z_2 \equiv$ Experience

$n = 25$ portfolio managers

$$R = \begin{array}{c} \\ \end{array}\begin{array}{ccc} Y & Z_1 & Z_2 \end{array}$$

$$R = \begin{bmatrix} 1.0 & -.35 & .82 \\ -.35 & 1.0 & -.60 \\ .82 & -.60 & 1.0 \end{bmatrix}$$

a) $r_{YZ_1} = -.35 \Rightarrow$ Riskier managers obtain lower returns

$r_{YZ_2} = .82 \Rightarrow$ More experienced managers get higher return.

(Book says $-.82$ in problem, not in table)

b) $r_{YZ_1 \cdot Z_2} = R_{Y_1 Y_2 \cdot Z_2} \qquad Y_2 = Z_2$

$$R_{YY} = \begin{bmatrix} 1.0 & -.35 \\ -.35 & 1.0 \end{bmatrix} \qquad R_{ZY}' R_{ZZ}^{-1} R_{ZY} = \begin{bmatrix} .82 \\ -.60 \end{bmatrix} \left(\frac{1}{1.0}\right) \begin{bmatrix} .82 & -.60 \end{bmatrix}$$

$$\Rightarrow R_{ZY}' R_{ZZ}^{-1} R_{ZY} = \begin{bmatrix} .67 & -.49 \\ -.49 & .36 \end{bmatrix}$$

$$\Rightarrow R_{YY} - R_{ZY}' R_{ZZ}^{-1} R_{ZY} = \begin{bmatrix} 1 & -.35 \\ -.35 & 1.0 \end{bmatrix} - \begin{bmatrix} .67 & -.49 \\ -.49 & .36 \end{bmatrix}$$

$$= \begin{bmatrix} .33 & .14 \\ .14 & .64 \end{bmatrix}$$

$$r_{Y_1 Y_2 \cdot Z_2} = \frac{.14}{\sqrt{.33}\sqrt{.64}} = \frac{.14}{.46} = .30 \Rightarrow$$

Controlling for Experience higher risk $\Rightarrow$ higher return

Note: $r_{Y_1 Y_2 \cdot Z_2} = \dfrac{r_{Y_1 Y_2}}{\sqrt{(1 - r_{Y_1 Z_2}^2)}\sqrt{(1 - r_{Y_2 Z_2}^2)}}$