

## Discrete probability distributions (random variables)

Distribution	$P(X=x)$	Expectation $\mu$	Variance $\sigma^2$	Example of application
Binomial $B(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$ $x=0, 1, \dots, n$	$np$	$np(1-p)$	The frequency in $n$ independent trials has a binomial distribution. Probability in each trial = $p$
Geometric $G(p)$	$(1-p)^{x-1} p$ $x=1, 2, 3, \dots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	The number of required trials until an event with probability $p$ occurs has a geometric distribution
Poisson $P(\lambda)$	$e^{-\lambda} \lambda^x / x!$ $x=0, 1, 2, \dots$	$\lambda$	$\lambda$	Distribution of number of points in random point process under certain simple assumptions. Approximation to the binomial distribution when $n$ is large and $p$ is small, $\lambda = np$ .
Hypergeometric $H(N, n, p)$	$\frac{\binom{Np}{x} \binom{N-Np}{n-x}}{\binom{N}{n}}$	$np$	$np(1-p) \frac{N-n}{N-1}$	This distribution is used in connection with sampling without replacement from a finite population with elements of two different kinds
Negative binomial or Pascal $NB(r, p)$	$\binom{x-1}{r-1} p^r (1-p)^{x-r}$ $x=r, r+1, \dots$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	The number of required trials until an event with probability $p$ occurs for the $r^{\text{th}}$ time has a negative binomial distribution

## Continuous probability distributions (random variables)

Distribution	$f(x)$	Expectation $\mu$	Variance $\sigma^2$	Example of application
Uniform $U(a, b)$	$\frac{1}{b-a}$ $a \leq x \leq b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	Certain waiting times Rounding off errors
Exponential $E(\lambda)$	$\lambda e^{-\lambda x}$ $x \geq 0$	$1/\lambda$	$1/\lambda^2$	Distribution of length of life when no aging
Normed normal distribution $N(0, 1)$	$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$	0	1	If $X$ has a general normal distribution, then $(X-\mu)/\sigma$ has a normed normal distribution
General normal distribution $N(\mu, \sigma)$	$\frac{1}{\sigma} \varphi\left(\frac{x-\mu}{\sigma}\right)$	$\mu$	$\sigma^2$	Under general conditions, the sum of a large number of random variables is approximately normally distributed ( <i>the central limit theorem</i> )
Gamma $\Gamma(n, \lambda)$	$\frac{\lambda^n}{\Gamma(n)} x^{n-1} e^{-\lambda x}$	$\frac{n}{\lambda}$	$\frac{n}{\lambda^2}$	Distribution of the sum of $n$ independent random variables with an exponential distribution with parameter $\lambda$
$\chi^2$ $\chi^2(r)$	$\frac{1}{2^{r/2} \Gamma(\frac{r}{2})} x^{\frac{r}{2}-1} e^{-\frac{x}{2}}$ $x \geq 0$ The parameter $r$ is called the "number of degrees of freedom"	$r$	$2r$	Distribution of $u_1^2 + u_2^2 + \dots + u_r^2$ , where $u_1, u_2, \dots, u_r$ are independent and have a normed normal distribution
$t$ $t(r)$	$\frac{a_r}{b_r} \left(1 + \frac{x^2}{r}\right)^{-\frac{r+1}{2}}$ $a_r = \Gamma\left(\frac{r+1}{2}\right)$ $b_r = \sqrt{r\pi} \Gamma\left(\frac{r}{2}\right)$	$0, r > 1$	$\frac{r}{r-2}, r > 2$	Distribution of $u/\sqrt{X/r}$ where $u$ and $X$ are independent, $u$ has a normed normal distribution and $X$ a $\chi^2$ -distribution with $r$ degrees of freedom
$F$ $F(r_1, r_2)$	$\frac{a_r x^{(r_1/2)-1}}{b_r (r_2 + r_1 x)^{\frac{r_1+r_2}{2}}}$ $x \geq 0$ $a_r = \Gamma\left(\frac{r_1+r_2}{2}\right) r_1^{r_1/2} r_2^{r_2/2}$ $b_r = \Gamma\left(\frac{r_1}{2}\right) \Gamma\left(\frac{r_2}{2}\right)$	$\frac{r_2}{r_2-2}$ $r_2 > 2$	$\frac{2r_2^2(r_1+r_2-2)}{r_1(r_2-2)^2(r_2-4)}$ $r_2 > 4$	Distribution of $(X_1/r_1)/(X_2/r_2)$ where $X_1$ and $X_2$ are independent and have $\chi^2$ -distributions with $r_1$ and $r_2$ degrees of freedom

Distribution	$f(x)$	Expectation $\mu$	Variance $\sigma^2$	Example of application
Beta $\beta(p, q)$	$a_{p,q} x^{p-1} (1-x)^{q-1}$ $0 \leq x \leq 1$ $a_{p,q} = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)}$ $p > 0, q > 0$	$\frac{p}{p+q}$	$\frac{pq}{(p+q)^2 (p+q+1)}$	Useful as apriori distribution for unknown probability in Bayesian models and in PERT-analysis
Weibull $W(\lambda, \beta)$	$\lambda^\beta \beta x^{\beta-1} e^{-(\lambda x)^\beta}$ $x \geq 0$ $F(x) = 1 - e^{-(\lambda x)^\beta}$	$\frac{1}{\lambda} \Gamma\left(1 + \frac{1}{\beta}\right)$	$\frac{1}{\lambda^2} (A - B)$ $A = \Gamma\left(1 + \frac{2}{\beta}\right)$ $B = \Gamma^2\left(1 + \frac{1}{\beta}\right)$	Useful as length of life distribution in reliability theory
Rayleigh $R(\sigma)$	$\frac{x}{\sigma^2} e^{-x^2/2\sigma^2}$ $x \geq 0$	$\sigma\sqrt{\pi/2}$	$2\sigma^2\left(1 - \frac{\pi}{4}\right)$	Useful in communications systems and in reliability theory
Cauchy $C(a)$	$\frac{a}{\pi(a^2 + x^2)}$	Does not exist	Does not exist	<p>If angle <math>\varphi</math> has the <math>U(-\pi/2, \pi/2)</math> distribution, then <math>a \tan \varphi</math> has the <math>C(a)</math> distribution</p> 