Data

amount of rain from 1960 to 1964 (p.383 problem 42)

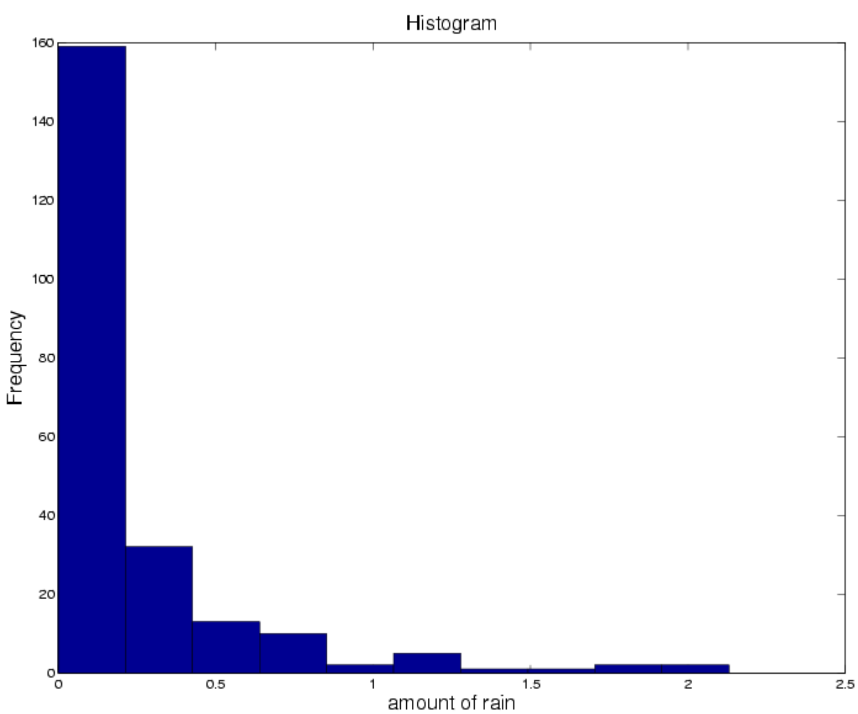
.02, 0.001, 0.001, 0.12, 0.08, 0.42, 1.72, 0.05, 0.01,…

n = 227

Goal

Construct a model to describe the data

Histogram



Model

Gamma distribution is a natural model

Gamma has two parameters: α, λ

Estimation

MME

$$\hat{\alpha}^{MME} = \frac{(\overline{X})^2}{\sum_{i=1}^{n}(X_i - \overline{X})^2 / n} = 0.3779$$

$$\hat{\lambda}^{MME} = \frac{\overline{X}}{\sum_{i=1}^{n}(X_i - \overline{X})^2 / n} = 1.6842$$

MLE

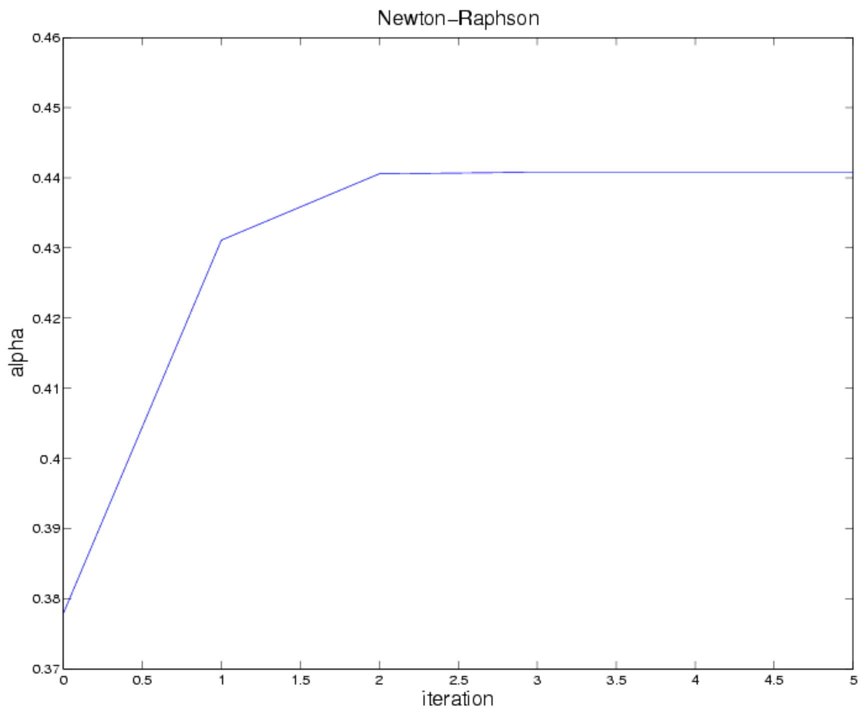
$$\hat{\lambda}^{MLE} = \frac{\hat{\alpha}^{MLE}}{\overline{X}}$$

$$\ell'(\alpha) = n \log \frac{\alpha}{\overline{X}} - n \frac{\Gamma'(\alpha)}{\Gamma(\alpha)} + \sum_{i=1}^{n} \log X_i = 0$$

Newton-Raphson

$$\hat{\alpha}^{(k+1)} = \hat{\alpha}^{(k)} - \frac{\ell'(\hat{\alpha}^{(k)})}{\ell''(\hat{\alpha}^{(k)})}, \quad \hat{\alpha}^{(0)} = \hat{\alpha}^{MME} = 0.3779$$

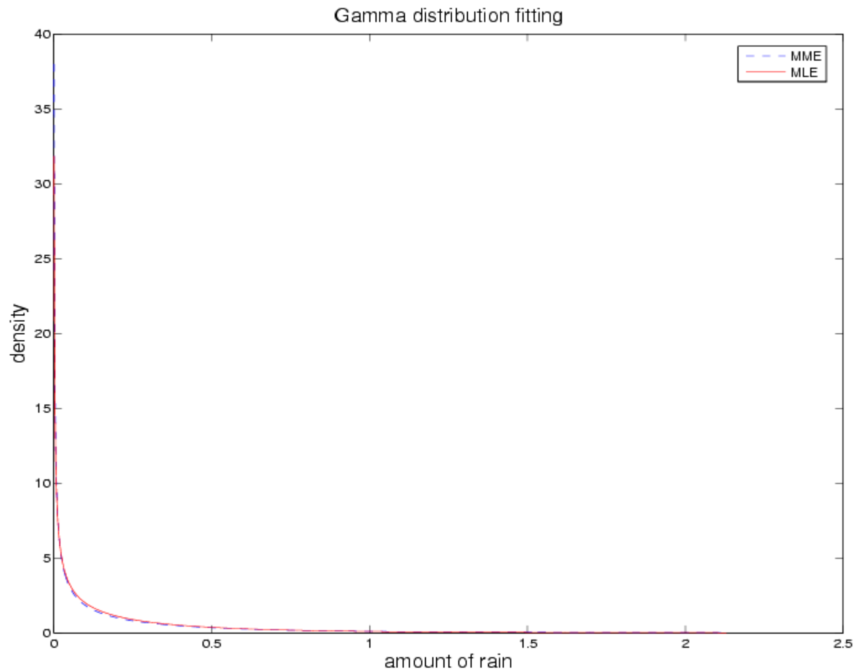
$$\varepsilon = 0.0000001$$

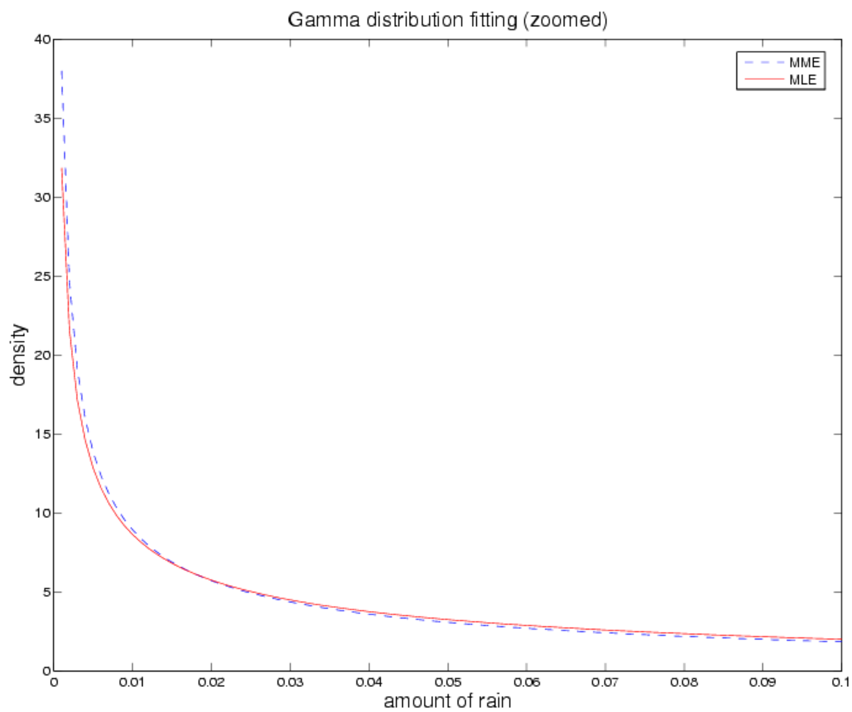


$$\hat{\alpha}^{MLE} = 0.4408$$

$$\hat{\lambda}^{MLE} = 1.9644$$

Distribution fitting





Usage: simulation (Bootstrap)

Generate simulated data from the fitted distribution

Sampling distribution: Bootstrap (MME)

Idea: To obtain an approximated distribution of estimate.

(e.g. histogram)

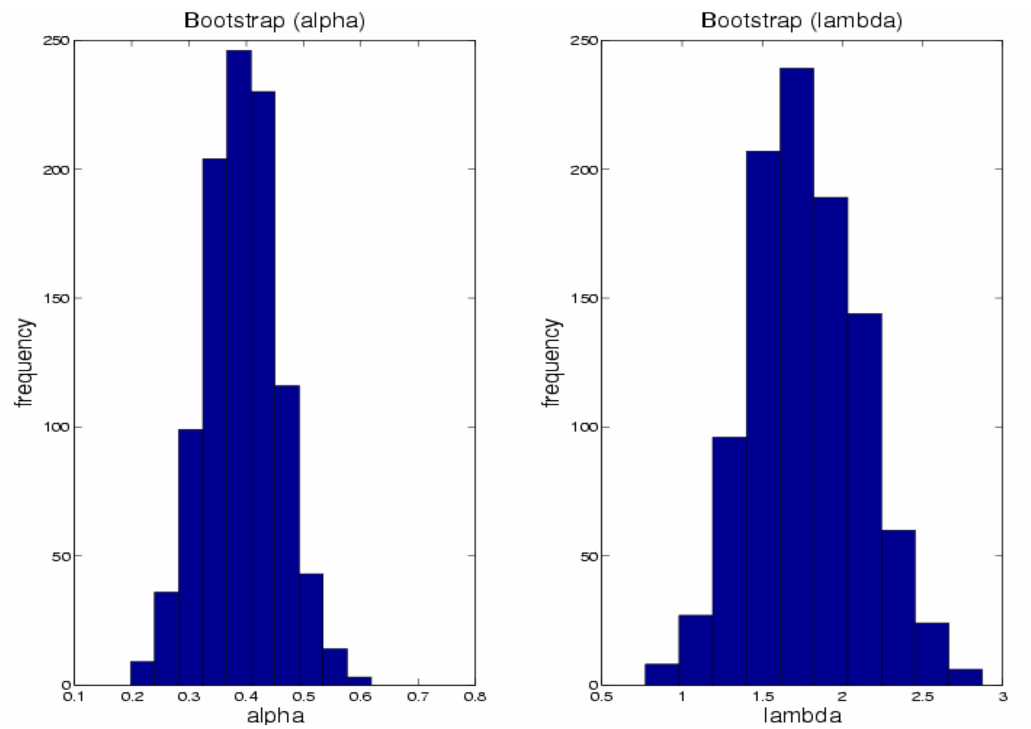
Problem: Need many observations, but we have only one.

Solution: Generate many samples using the fitted distribution and obtain many estimates.

Generate B=1000 samples (size n=227) from Gamma($\hat{\alpha}^{MME}, \hat{\lambda}^{MME}$)

For each sample j=1,…,1000, calculate ($\hat{\alpha}_j^{*MME}$, $\hat{\lambda}_j^{*MME}$)

Plot histograms of $\hat{\alpha}^{*MME}$ and $\hat{\lambda}^{*MME}$



95% Confidence Interval

$$\hat{\theta}^{MME} \pm 1.96\sqrt{Var(\hat{\theta}^{MME})}$$

$$\hat{S}_{\hat{\alpha}} = \sqrt{\frac{1}{B}\sum_{j=1}^{B}(\hat{\alpha}_j^{*MME} - \overline{\alpha})^2}\,, \quad \overline{\alpha} = \frac{1}{B}\sum_{j=1}^{B}\hat{\alpha}_j^{*MME}$$

$$\hat{S}_{\hat{\lambda}} = \sqrt{\frac{1}{B}\sum_{j=1}^{B}(\hat{\lambda}_j^{*MME} - \overline{\lambda})^2}\,, \quad \overline{\lambda} = \frac{1}{B}\sum_{j=1}^{B}\hat{\lambda}_j^{*MME}$$