Formula Sheet: Exam 1

$$\overline{X} = \frac{\sum X_i}{n} \qquad \sigma_x = \sqrt{\frac{\sum (x_i - \overline{x})^2}{n}} = \sqrt{\frac{\sum xi^2 - \frac{(\sum xi)^2}{n}}{n}} \qquad s_x = \sqrt{\frac{\sum (x_i - \overline{x})^2}{n-1}} = \sqrt{\frac{\sum xi^2 - \frac{(\sum xi)^2}{n}}{n-1}}$$

Simple Linear Regression

Model: $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$

$$S_{XX} = \sum (x_i - \overline{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} \qquad S_{YY} = \sum (y_i - \overline{y})^2 = \sum y_i^2 - \frac{(\sum y_i)^2}{n}$$

$$S_{XY} = \sum (x_i - \overline{x})(y_i - \overline{y}) = \sum x_i y_i - \frac{\sum x_i \sum y_i}{n}$$

$$b_0 = \overline{y} - b_1 \overline{x}$$

$$\hat{y} = b_0 + b_1 x$$

$$\hat{\mu} = b_0 + b_1 x$$

$$MSM = \frac{\sum (\hat{y}_i - \overline{y})^2}{1}$$

$$s^2 = MSE = \frac{\sum (yi - \hat{y}_i)^2}{n-2} \qquad b_1 = \frac{Sxy}{Sxx} = r\frac{s_y}{s_x} = r\sqrt{\frac{Syy}{Sxx}}$$

$$MST = \frac{\sum (yi - \overline{y})^2}{n-1} \qquad SSE = \sum (y_i - \hat{y}_i)^2 = \sum e_i^2$$

$$SE_{b_0} = s\sqrt{\frac{1}{n} + \frac{\overline{x}^2}{\sum (xi - \overline{x})^2}} \qquad SE_{\hat{y}} = s\sqrt{1 + \frac{1}{n} + \frac{(x^* - \overline{x})^2}{\sum (xi - \overline{x})^2}}$$

$$R^2 = \frac{SSM}{SST} \qquad r = \frac{Sxy}{\sqrt{SxxSyy}}$$

Test	Statistic	Dist.
$H_0: \beta_1 = 0$	$F = \frac{MSM}{MSE}$	$F_{1,\,n-2}$
$H_0: \beta_i = 0$	$t = \frac{b_i}{SE_{b_i}}$	t_{n-2}
$H_0: \rho = 0$	$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$	t_{n-2}

$$SE_{\hat{\mu}} = s\sqrt{\frac{1}{n} + \frac{(x^* - \overline{x})^2}{\sum (xi - \overline{x})^2}}$$

$$SE_{b_1} = \frac{s}{\sqrt{\sum (xi - \overline{x})^2}}$$

Multiple Linear Regression

Model: $y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_p x_{ip} + \varepsilon_i$

$$SSE = \sum (y_i - \hat{y}_i)^2 = \sum e_i^2 \qquad s^2 = MSE = \frac{\sum (y_i - \hat{y}_i)^2}{n-(p+1)}$$

$$MSM = \frac{\sum (\hat{y}_i - \overline{y})^2}{p}$$

$$MST = \frac{\sum (yi - \overline{y})^2}{n-1} \qquad R^2 = \frac{SSM}{SST}$$

Test	Statistic	Dist.
$H_0: \beta_i = 0$	$t = \frac{b_i}{SE_{b_i}}$	$t_{n-(p+1)}$
$H_0: \beta_1 = \beta_2 = \ldots = \beta_p = 0$	$F = \frac{MSM}{MSE}$	$F_{p,\,n-(p+1)}$

Formula Sheet: Exam 2

One-Way ANOVA

Model: $x_{ij} = \mu_i + \varepsilon_{ij}$ i=1, 2, ..., I j=1,2,...,ni

$$\bar{x}i = \frac{1}{ni}\sum_{j=1}^{ni} xij \qquad Sp^2 = \frac{(n_1-1)s_1^{\,2} + (n_2-1)s_2^{\,2} + \ldots + (n_I-1)s_I^{\,2}}{(n_1-1)+(n_2-1)+\ldots+(n_I-1)}$$

$$eij = xij - \bar{x}i$$

$$\bar{x} = \frac{1}{I}\sum_{i=1}^{I} \bar{x}i \qquad R^2 = SSG / SST$$

$$tij = \frac{\bar{x}i - \bar{x}j}{Sp\sqrt{\frac{1}{ni} + \frac{1}{nj}}} \qquad MSD = t^{**}Sp\sqrt{\frac{1}{ni} + \frac{1}{nj}}$$

$$(\bar{x}i - \bar{x}j) \pm t^{**}Sp\sqrt{\frac{1}{ni} + \frac{1}{nj}}$$

$$c \pm t^{*}SEc \qquad df: N - I$$

$$MSG = \frac{\sum_{i=1}^{I} ni(\bar{x}i - \bar{x})^2}{I-1} \qquad MSE = \frac{\sum_{i=1}^{I}(ni-1)si^2}{N-I}$$

$$MST = \frac{\sum_{obs}(xij - \bar{x})^2}{N-1}$$

$$\psi = \sum ai\,\mu i$$

$$c = \sum ai\,\bar{x}i$$

$$SEc = Sp\sqrt{\sum \frac{ai^{\,2}}{ni}}$$

Test	Statistic	Dist.
$Ho: \psi = 0$	$t = \frac{c}{SEc}$	$df: N - I$
$Ho: \mu 1 = \mu 2 = \ldots = \mu I$	$F = \frac{MSG}{MSE}$	$F_{I-1,N-I}$

Two-Way ANOVA

Model: $xijk = \mu ij + \varepsilon ijk$ i=1,2,...I j=1,2,...,J k=1,2,...,nij

$$\bar{x}ij = \frac{1}{nij}\sum_{k=1}^{nij} xijk \qquad Sp^2 = \frac{\sum(nij-1)sij^2}{\sum(nij-1)}$$

$$MSA = \frac{SSA}{I-1} \quad MSB = \frac{SSB}{J-1} \quad MSAB = \frac{SSAB}{(I-1)(J-1)} \quad MSE = \frac{SSE}{N-IJ} \quad MST = \frac{SST}{N-1}$$

$$Fa = \frac{MSA}{MSE} \qquad Fb = \frac{MSB}{MSE} \qquad Fab = \frac{MSAB}{MSE}$$

$$F_{I-1,N-IJ} \qquad F_{J-1,N-IJ} \qquad F_{(I-1)(J-1),N-IJ}$$

Formula Sheet: Exam 3

Two-Way Tables/Categorical

$$e_{ij} = \frac{r_i total \times c_j total}{n}$$

$$X^2 = \sum \frac{\left(x_{ij} - e_{ij}\right)^2}{e_{ij}}$$

Compare to $\chi^2_{(r-1)(c-1)}$

Nonparametrics

$$\mu_W = \frac{n_1(N+1)}{2}$$

$$\sigma_W = \sqrt{\frac{n_1 n_2 (N+1)}{12}}$$

$$Z = \frac{W - \mu_W}{\sigma_W}$$

$$\mu_{W_+} = \frac{n(n+1)}{4}$$

$$\sigma_{W_+} = \sqrt{\frac{n(n+1)(2n+1)}{24}}$$

$$Z = \frac{W_+ - \mu_{W_+}}{\sigma_{W_+}}$$

$$H = \frac{12}{N(N+1)} \sum \frac{R_i^2}{n_i} - 3(N+1)$$

Compare to $\chi^2_{(I-1)}$