Exercise Set 4 - Solution

1. (a) $P = 15000\left(\frac{1}{1.06}\right)^2 + 25000\left(\frac{1}{1.06}\right)^5$

$= 32{,}031.40$

(b) $\bar{d} = \dfrac{2(15000)(1.06)^{-2} + 5(25000)(1.06)^{-5}}{32{,}031.40}$

$= 3.7496694$

$\bar{v} = v\bar{d} = \left(\frac{1}{1.06}\right)(\bar{d}) = 3.537424$

$\bar{c} = \dfrac{2(3)(15000)(1.06)^{-4} + 5(6)(25000)(1.06)^{-7}}{32{,}031.40}$

$= 17.7975757$

(c) $P(.061) \doteq P(.06)\left[1 - (.001)(\bar{v}) + \frac{(.001)^2}{2}\bar{c}\right]$

$= [32{,}031.40]\left[1 - (.001)(3.537424) + (.0000005)(17.7975757)\right]$

$\doteq 31{,}918.38$

2. At the end of year two, the two-year bond returns

$$2000(1 + 1.25r) \overset{set}{=} 2L \qquad (*)$$

At the end of year one, the one-year bond returns

$$800(1+r)$$ that plus the coupon from the two-year bond must equal L, ie

$$800(1+r) + 2000(1.25r) \overset{set}{=} L$$

or $$800 + r(3300) = L$$

Multiply by 2 and set it equal to (*) above

$$2000 + 2500r = 1600 + r(6600)$$

$$4100r = 400$$

$$r = .097561$$

3. X = amt invested today in 1-yr zero coupon bond

Y = " " " " 3-yr " " "

Returns $R_1 = X(1.08) \quad R_2 = -1000$

$R_3 = Y(1.1)^3$

$$P(i) = X(1.08)v - 1000v^2 + (1.1)^3 Y v^3$$

$$P'(i) = -X(1.08)v^2 + 2000v^3 - 3(1.1)^3 Y v^4$$

$$P''(i) = 2X(1.08)v^3 - 6000v^4 + 12(1.1)^3 Y v^5$$

Use $i_0 = .1$ and set

$P(i_0) = 0$ and $P'(i_0) = 0$ producing

$$X\left(\frac{1.08}{1.1}\right) - 1000\left(\frac{1}{1.1}\right)^2 + Y = 0$$

and $-X\left(\frac{1.08}{(1.1)^2}\right) + 2000\left(\frac{1}{1.1}\right)^3 - \frac{3}{1.1}Y = 0$

Substitute the first into the second $\times(1+i)$ produces

$$-X\left(\frac{1.08}{1.1}\right) + 2000\left(\frac{1}{1.1}\right)^2 - 3\left[-X\left(\frac{1.08}{1.1}\right) + 1000\left(\frac{1}{1.1}\right)^2\right] = 0$$

$2X\left(\frac{1.08}{1.1}\right) = 1000\left(\frac{1}{1.1}\right)^2$ or $X_0 = 420.88$

and $Y_0 = 1000\left(\frac{1}{1.1}\right)^2 - X_0\left(\frac{1.08}{(1.1)}\right) = 413.22$

Then checking

$$P''(i_0) = 2(420.88)(1.08)\left(\frac{1}{1.1}\right)^3 - 6000\left(\frac{1}{1.1}\right)^4 + 12(1.1)^3(413.22)\left(\frac{1}{1.1}\right)^5$$

$$= 683.02 - 4{,}098.08 + 4{,}098.05 > 0$$

confirms Redington immunization at $i_0 = .1$