

Forward Contract An agreement between two parties to buy (sell) an asset (commodity) at a specific time in the future.

It will specify

- (a) the asset to be transferred
- (b) the day and time of delivery
- (c) the price to be paid

It obligates both buyer and seller to perform this transaction.

The date of the transaction is T , the (expiry) expiration date of the agreement.

an unknown future value,

Typically no premium is paid to create this agreement.

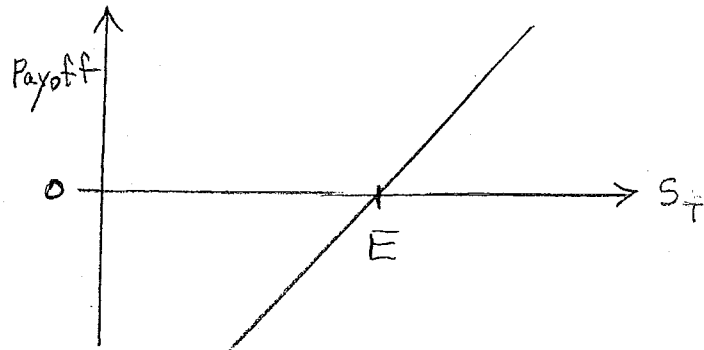
Example: The owner of XYZ stock currently valued at \$40 per share, agrees to sell 1000 shares at \$42 per share one year from today.

The buyer in a forward contract is said to have the long position in the agreement. The seller has the short position.

Payoffs (Inherent Values)

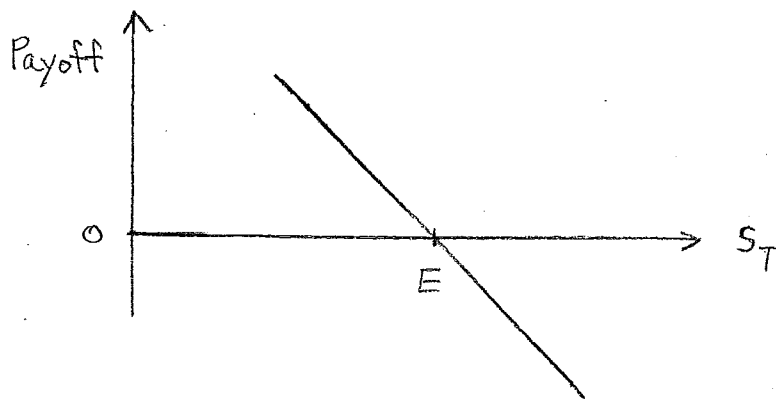
Let E denote the price that the buyer agreed to pay for the asset at expiry. Then

$$\begin{aligned} \text{payoff (long forward)} &= S_T - E \\ &= (\text{value of asset @ expiry}) - (\text{forward price}) \end{aligned}$$



Likewise, the payoff of the short position in this agreement is :

$$\text{payoff (short forward)} = E - S_T$$



A profit diagram is a cost adjusted payoff diagram at expiry:

$$\text{profit} =$$

In the case of a forward contract, no premium is paid to establish the position. Therefore payoff and profit are identical.

Example: (Index based forward contract)

Suppose party A takes the long position relative to the Dow Jones Average being \$17,000 three months from today with party B in the short position.

If at expiry, the DJA is at \$17,680, then party B pays party A \$680.

If at expiry, the DJA is \$16,640, then A pays B \$360.

This is a cash settlement deal since no actual asset changes hands.

Example: Consider the purchase of a \$5000 zero coupon bond with a maturity date of one year (expiry). The position of the purchaser at expiry produces a payoff = \$5000.

This is a constant payoff function since there is no asset of unknown future value involved. Assuming a 4% annual interest rate, the purchase price of the bond was

$$\begin{aligned} \text{price at beginning} &= \frac{5000}{(1.04)} \\ &= 4807.69 \end{aligned}$$

D2-5B

Exercise D2-A Alan enters into a long forward contract. If the spot price at expiry were S , his payoff is $-\$20$. If the spot price was 10% higher, his payoff is $\$5$. Determine S .

Exercise D2-B You are given:

- (i) Spot price of a market index today is \$1,300
- (ii) Forward price of a one-year forward contract is \$1,340.
- (iii) A one-year zero coupon bond maturing at \$100 sells today for \$97.50.
- (iv) Spot price of the market index one year from today is \$1365.

Compare the profits one-year from today for a long index strategy versus a long forward strategy.

The profit of this agreement is D2-6

payoff - (future value of investment)

$$= 5000 - (4807.69)(1.04) = \underline{0}$$

In a sense then, a zero coupon bond is the standard whereby profits are judged. A positive profit means the agreement is more profitable than a zero coupon bond, which has a constant 0 profit.

Call Options

A call option gives the buyer the

price at a fixed time in the future (European-style).

(a) The fixed time is the expiration date (expiry, T)

(b)

(c) The buyer is said to exercise the option when choosing to pay the strike price at expiry.

The option buyer must pay the option seller (option writer) a _____ because at expiry the buyer (not the seller) has the choice whether or not to execute the option.

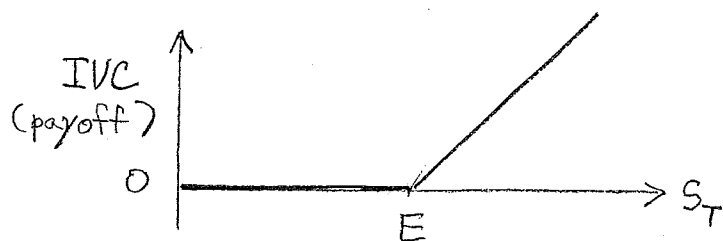
Let C denote the future value of this premium at expiry, i.e. the value of

$$C = (\text{premium})(1+i)$$

where i is the risk free interest to expiry.

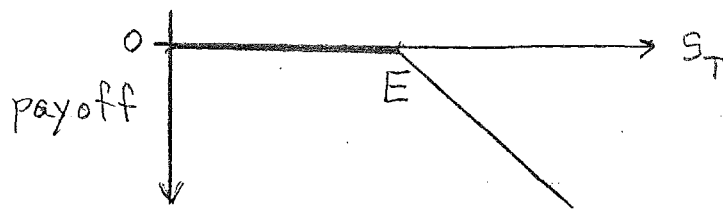
The payoff at expiry for the option buyer, the long position in this agreement is

the advantage of having the agreement. If the value of the asset S_T is below the strike price E , the long position will simply not exercise the option. If S_T exceeds E , the option will be exercised. This payoff is often described as the inherent value of the call (IVC).



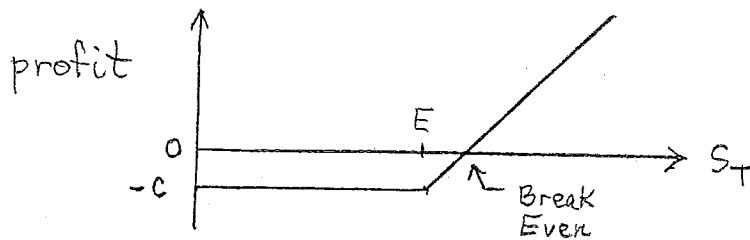
Long
Call

The payoff at expiry for the option writer (seller), the _____ in this agreement is just the opposite.

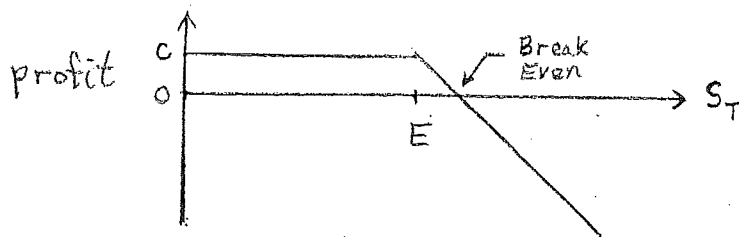


Short
Call

But neither of these diagrams take into account the (future value) of the premium C . The profit (@ expiry) diagrams do.



Long
Call



Short
Call

Recall that

We consider only European-style options which have an exercise point at expiry. Other styles exist, e.g. the American-style can be exercised at any time up to expiry.

Exercise D2-C You write a one-year call option with a strike price of E and a premium of \$10.50. Your profit at maturity is 0 at the spot price of \$120.15. The risk free interest rate is 5%. Find E .

Exercise D2-D

You buy a 6 month call option with a strike price of \$125 at a premium of \$5.25. Six months later the asset has a value of \$130.62 and you make a profit of \$0.27. What is the risk-free rate of interest?

Put Options

Buying a Put Option gives the buyer the

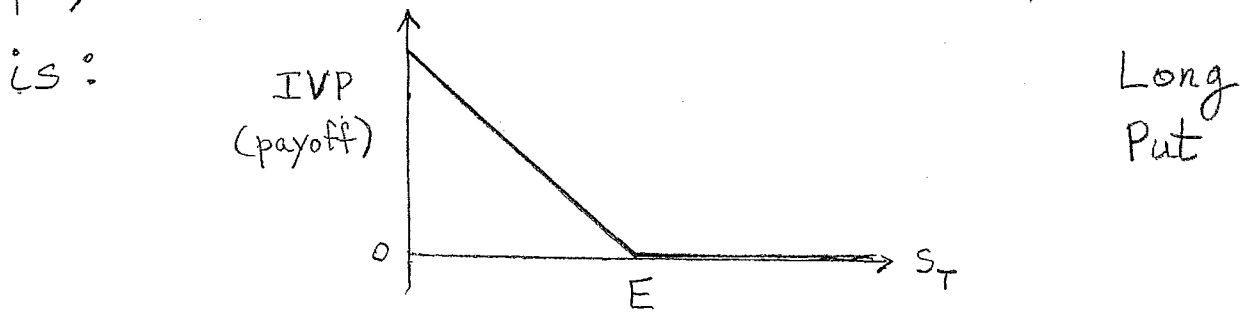
The option buyer is described as in the long position in the agreement, but the short position (seller) relative to the asset.

The other party in the agreement, the option writer, will be paid a premium to incentivise the transaction. The option writer is in the short position in the agreement, but the long position (buyer) relative to the asset.

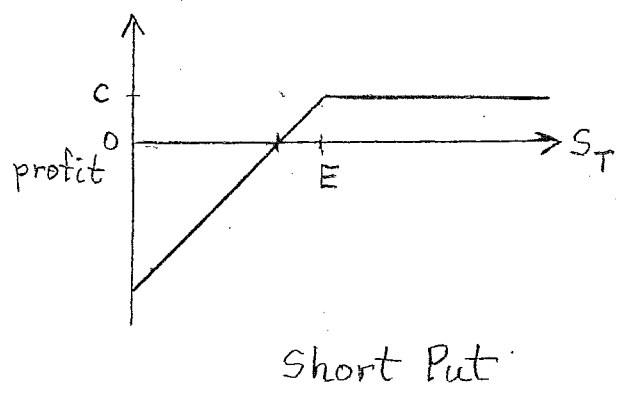
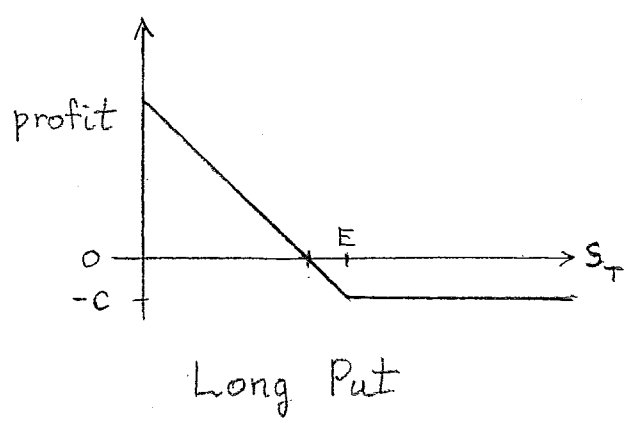
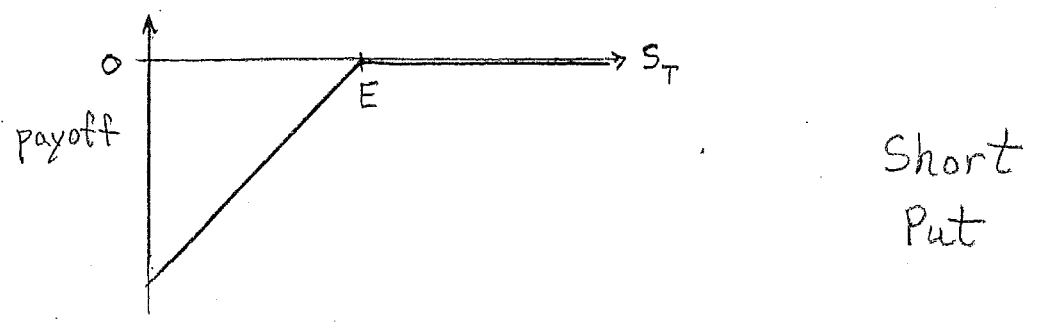
The payoff of the long position in a put is

$$\text{payoff}(\text{long}) = \begin{cases} E - S_T & \text{if } S_T < E \\ 0 & \text{if } S_T \geq E, \end{cases}$$

ie the option will not be exercised if the asset's value S_T at expiry exceeds the strike price E . The graph of this payoff (inherent value of the put, IVP) is:



The payoff of the short position in the put agreement is the opposite,



Payoff vs Profit

$$\text{Profit}_T = \text{Payoff}_T$$

$$= \text{Payoff}_T$$

+ (future value at T of all payments received at $t=0$ and all assets borrowed at $t=0$)

All profit diagrams are comparisons of the payoff to investing the expenses (all money spent and assets committed) in a zero-coupon bond maturing at time T with a risk-free interest rate,

Exercise D2-E A fire insurance policy covers a house worth \$280,000. The policy has a \$2,000 deductible and a premium of \$780. The payoff on the policy is expressed in the form of a purchased put as $\max[0, X - (\text{value of house after fire})]$ with X the strike price. Find X .

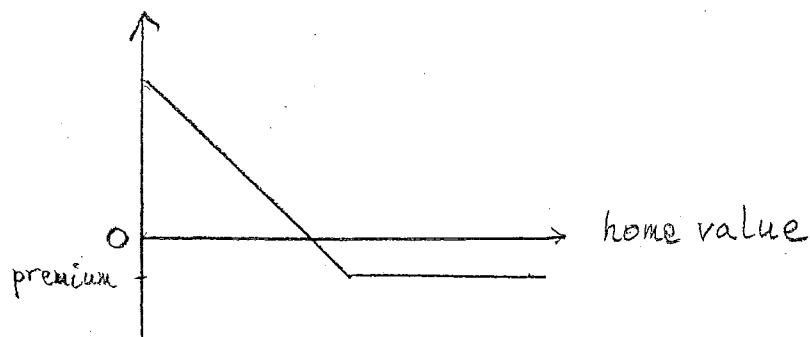
D2-12C

Exercise D2-F You write a one-year put option with a strike price of \$220 for a premium of \$12.50. If the risk-free rate of interest is 4.5% annual effective, what is the largest and smallest profit that you can make on this agreement at the expiration.

Options as Insurance

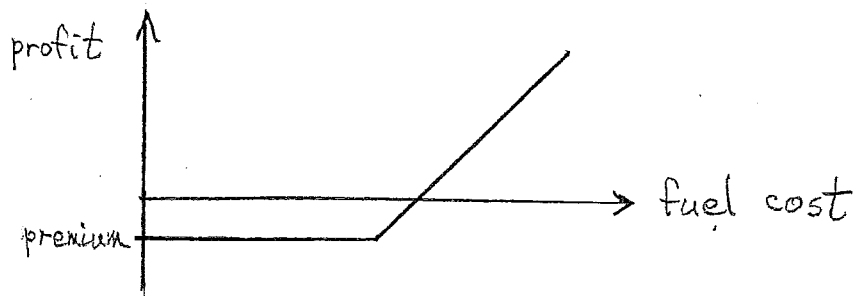
Particularly when the asset's value is varying because of a volatile market or when its value could rise or fall precipitously under adverse conditions,

Example: Homeowners insurance is like buying a long put position. If the value of the home falls due to storm, fire, etc., then the homeowner is compensated for the loss by the insurance company. For protection against this potential loss in value, the homeowner pays a premium. The profit diagram for the homeowner resembles:



Similarly, a long call could be interpreted as buying insurance against a rising market.

Example: A company which uses substantial amounts of diesel fuel might pay a premium for an agreement that will compensate them if the cost of diesel fuel exceeds an upper threshold which they can not afford to exceed. Their profit diagram for their position in the agreement would look like:



Other Issues in Options

When buying stock via an option, it is necessary to consider which party is to receive stock dividends paid between the beginning of the agreement and expiry. Typically the value of a stock will drop by the dividend amount right after the dividend is paid.

An option must be exercised. Some are automatically exercised at the appropriate time, while others are not.

An option writer may be required to post collateral, to insure that the writer will not default if the asset value is unfavorable.

There may be a tax advantage to using a well designed option, as special tax rules may apply.

Exercise D2-1 Alice enters into a one-year forward with a \$200 forward price. At the same time, she buys a zero-coupon bond that will mature at the forward price at a interest rate of 4% effective. If the spot price at expiry is \$220, find the profit Alice makes.

Exercise D2-2 Person A enters into a one-year forward contract with a forward price of \$150. Person B enters into a one-year short forward contract for a different asset with a forward price of \$175. The spot price of both assets at expiry is \$170. Find the sum of their profits.

Exercise D2-3 Which of the following statements are true about a forward contract?

- (I) When the contract is entered into, one party pays the other party the forward price.
- (II) The party who is obligated to buy the underlying asset at expiration has a long position in the asset.
- (III) The sum of the payoffs to the two parties is 0 for any spot price at expiration.

Exercise D2-4 Person A pays a premium of \$8 to person B for a one-year call with a strike price of \$65. The risk-free interest rate is 6% effective. What is the spot price at expiration if person B's profit is 0?

Exercise D2-5 Which of the following parties has the long position in the asset?

- (I) Writer of a call
 - (II) Party obligated to sell the asset in a forward contract
 - (III) The party who buys the asset outright.
-

Exercise D2-6 You buy a 100-strike call on asset A and sell a 92-strike call on asset B, both one-year calls. What is your combined payoff from these transactions, if both assets have a spot price of \$96 at expiration.

Exercise D2-7 You write a 2-year put with a strike price of X at a premium of $\$10.40$. The risk-free rate of interest is 3.5% annual effective. Your profit at a spot price of $\$98$ is $\$5.50$. Find X .

Exercise D2-8 You buy a 6 month put with a strike price of $\$80$ for a premium of $\$5.75$. You also write a 6-month $\$95$ -strike put for $\$9.70$ on the same asset. The risk-free rate of interest is 4.5% annual effective. The spot price at expiration is $\$88$. Find the total profit you make in these transactions.

Exercise D2-9 Auto insurance has a deductible of $\$1000$ and a premium of $\$1,200$. Assume the risk-free rate of interest is 3.5%. Find the customer's profit when the auto has $\$2,355$ damage right at the end of the policy year.