STA 4183 Third Exam - Fall 2016 Name: Key

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SHOW YOUR WORK AND REASONING ON ALL QUESTIONS

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An investment account entered the year 2015 with a balance of \$2000. Three months after January 1, the balance was \$2020, just before a deposit of of \$400 was made. Eight months after January 1, the balance was \$2510, just before a withdrawal of \$200 was made. The investment finished the year 2015 with a balance of \$2390.

(a) Find the dollar weighted (simple interest) yield of this investment for the year 2015. (8 pts)

$A = 2000 \quad B = 2390 \quad C = 400 - 200 = 200$

$I = B - A - C = 2390 - 2000 - 200 = 190$

$$i = \frac{I}{A + \sum C_t(1-t)} = \frac{190}{2000 + 400(3/4) - 200(1/3)}$$

$$= \frac{190}{2233.33} = .085074627$$

(b) Find the time weighted yield of this investment for the year 2015. (8 pts)

$$(1+i) = \left(\frac{2020}{2000}\right)\left(\frac{2510}{2020+400}\right)\left(\frac{2390}{2510-200}\right)$$

$$= \left(\frac{2020}{2000}\right)\left(\frac{2510}{2420}\right)\left(\frac{2390}{2310}\right)$$

$$= 1.083841186$$

$$i = .083841186$$

i	B_i	C_i
0	2000	
1	2020	400
2	2510	-200
3	2390	

(c) What is the difference in emphasis between these two yield computations, ie when is one preferred over the other? (2 pts)

Time weighted puts more emphasis on the fund administrators' performance

Dollar weighted puts more emphasis on the contributions of the investor.

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An investment account entered the year 2015 with a balance of \$2000. Two months after January 1, the balance was \$2020, just before a deposit of of \$400 was made. Nine months after January 1, the balance was \$2510, just before a withdrawal of \$200 was made. The investment finished the year 2015 with a balance of \$2490.

(a) Find the dollar weighted (simple interest) yield of this investment for the year 2015. (8 pts)

$A = 2000 \qquad B = 2490 \qquad C = 400 - 200 = 200$

$I = B - A - C = 2490 - 2000 - 200 = 290$

$$i = \frac{I}{A + \Sigma C_t (1-t)} = \frac{290}{2000 + 400(5/6) - 200(1/4)}$$

$$= \frac{290}{2283.333} = .127007299$$

(b) Find the time weighted yield of this investment for the year 2015. (8 pts)

$$(1+i) = \left(\frac{2020}{2000}\right)\left(\frac{2510}{2020+400}\right)\left(\frac{2490}{2510-200}\right)$$

$$= \left(\frac{2020}{2000}\right)\left(\frac{2510}{2420}\right)\left(\frac{2490}{2310}\right)$$

$$= 1.12919019$$

$$i = .12919019$$

i	B_i	C_i
0	2000	
1	2020	400
2	2510	-200
3	2490	

(c) What is the difference in emphasis between these two yield computations, ie when is one preferred over the other? (2 pts)

Time weighted puts more emphasis on the fund administrator's performance

Dollar weighted puts more emphasis on the contributions of the investor.

An investment of \$1,000 today ($t = 0$) will yield returns of \$100 one year from today ($t = 1$), \$200 two years from today ($t = 2$) and \$900 three years from today ($t = 3$). The investment project ends at $t = 3$.

(a) Use your financial calculator to find the yield (internal rate of return) for this investment. Indicate your calculator entries. (8 pts)

$C_0 = -1000 \quad C_1 = 100 \quad C_2 = 200 \quad C_3 = 900$

IRR CPT

$i = .071139788$

(b) Now suppose any positive returns before $t = 3$ can only be reinvested at a 4% annual effective interest rate. Accumulating all the positive returns to $t = 3$, find the overall annual effective interest rate (yield) of this investment between $t = 0$ and $t = 3$. (8 pts)

At $t = 3$, the accumulation is

$$100(1.04)^2 + 200(1.04) + 900 = 1216.16$$

$$(1+i)^3 = \frac{1216.16}{1000} = 1.21616$$

$$(1+i) = 1.06740747$$

$$i = .06740747$$

Briefly answer each of the following:

(a) What does $\overline{d}$ measure? Describe it in words. (4 pts)

$\overline{d}$ is a weighted average of the times when returns are made, with weights equal to the PV of the respective return.

(b) What does it mean for a bond to be callable? (4 pts)

It means the bond can be redeemed at the discretion of the borrower (bond issuer).

(c) What does it mean for a bond to be purchased at premium? (4 pts)

It means the price (P) is larger than the redemption value (C).

(d) Consider a return of \$100 at the end of year 1 and a return of \$500 at the end of year 3. If the effective annual rate of interest is 5%, SET UP the computation of the volatility $\overline{v}$ for this series of returns. (YOU DO NOT HAVE TO COMPUTE IT.)

(5 pts)

$$\overline{v} = v\overline{d} = \left(\frac{1}{1.05}\right)\left[\frac{1\left(\frac{1}{1.05}\right)(100) + 3\left(\frac{1}{1.05}\right)^3 500}{\left(\frac{1}{1.05}\right)(100) + \left(\frac{1}{1.05}\right)^3 500}\right]$$

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Current annual spot rates are listed below.

Term Length t	Spot Rate
1	4.5%
2	5.5%
3	6.0%
4	6.5%
5	7.0%

(a) Find the first three one-year implied forward rates (ie $f_0, f_1,$ and f_2). (8 pts)

$f_0 = S_1 = .045$

$f_1 = .0650957$

$f_2 = .0700712$

$(1.045)(1+f_1) = (1.055)^2$ $\quad (1+f_1) = 1.0650957$

$(1.055)^2(1+f_2) = (1.06)^3$ $\quad (1+f_2) = 1.0700712$

(b) Find the three year deferred 2 year forward rate. (7 pts)

$(1+.06)^3(1+{}_2f_3)^2 = (1.07)^5$

$(1+{}_2f_3)^2 = 1.177609478 55$

$(1+{}_2f_3) = 1.0851771646$

${}_2f_3 = .0851771646$

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Current annual spot rates are listed below.

Term Length t	Spot Rate
1	3.5%
2	4.0%
3	4.8%
4	5.0%
5	5.5%

(a) Find the first three one-year implied forward rates (ie $f_0, f_1,$ and f_2). (8 pts)

$f_0 = s_1 = .035$

$f_1 = .045024$

$f_2 = .064185$

$(1.035)(1+f_1) = (1.04)^2$ $\quad (1+f_1) = 1.045024$

$(1.04)^2(1+f_2) = (1.048)^3$ $\quad (1+f_2) = 1.064185$

(b) Find the two year deferred 3 year forward rate. (7 pts)

$(1.04)^2(1+{}_3f_2)^3 = (1.055)^5$

$(1+{}_3f_2)^3 = 1.208357994$

$(1+{}_3f_2) = 1.06512$

${}_3f_2 = .06512$

An investment of \$4000 today produces payments of \$1000 two years from today and \$5000 four years from today.

(a) Find the net present value of this cash flow using an effective annual interest rate of 5%. (6 pts)

$$NPV = -4000 + 1000\left(\frac{1}{1.05}\right)^2 + 5000\left(\frac{1}{1.05}\right)^4$$

$$= -4000 + 907.02948 + 4113.51237$$

$$= \underline{1020.54}$$

(b) Find the annual effective yield (internal rate of return) for the cash flow described above. (You are expected to solve this by a direct method using the IRR definition, NOT by using a financial calculator.) (10 pts)

$$4000 = 1000v^2 + 5000v^4 \qquad v^2 = x$$

$$5x^2 + x - 4 = 0$$

$$(5x - 4)(x + 1) = 0$$

$$x = \frac{4}{5} = v^2 \qquad v = .894427191$$

$$(1+i) = 1.118034 \qquad \underline{i = .118034}$$

A \$20,000 par value 10-year bond with a 8% nominal coupon rate payable semi-annually will be redeemed at maturity for \$20,000. The price of the bond produces a nominal annual yield of 6% convertible semiannually.

(a) Find the price of this bond. Show your reasoning and the use of formulas. (10 pts)

$C = F = 20{,}000 \qquad r = g = .04$

$i = .03 \qquad n = 20$

$$P = Fr\,a_{\overline{n}|} + K$$

$$= (20000)(.04)\,a_{\overline{20}|.03} + 20000\left(\frac{1}{1.03}\right)^{20}$$

$$= 800(14.87747486) + 11{,}073.51508$$

$$= 11{,}901.979888 + 11{,}073.51508$$

$$= 22{,}975.49$$

(b) Calculate the amount of interest and the amount of principle in the 7th coupon payment. Use a compound interest bond amortization scheme. (8 pts)

$$P_7 = C(g-i)v^{n+1-t} = 20{,}000\,(.04-.03)\left(\frac{1}{1.03}\right)^{21-7}$$

$$= 20{,}000(.01)\left(\frac{1}{1.03}\right)^{14} = 132.22$$

$$I_7 = 800 - 132.22 = 667.78$$

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