Chapter 09 - More Advanced Financial Analysis

Section 9.4 - Recognition of Inflation

Interest rates are positively correlated with inflation and/or the expectation of inflation. Let

$i =$ the usual (nominal) rate of interest representing the investment dollar growth rate.

$r =$ the rate of inflation *and*

$i' =$ the real rate of interest, the growth in money that is over and above inflation.

Then

$$(1 + i) = (1 + i')(1 + r).$$

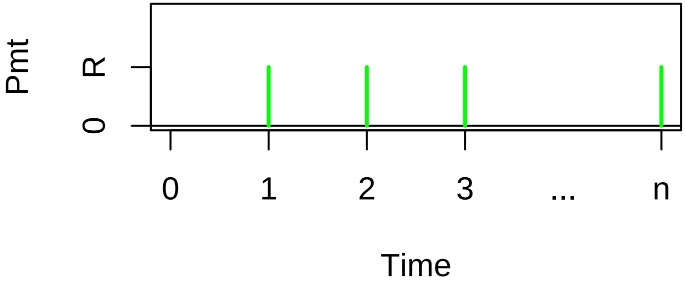
It follows that

$$i = i' + r + i'r,$$

Note that the quantity i' can be negative if $r > i$, in which case inflation exceed interest (dollar) growth.

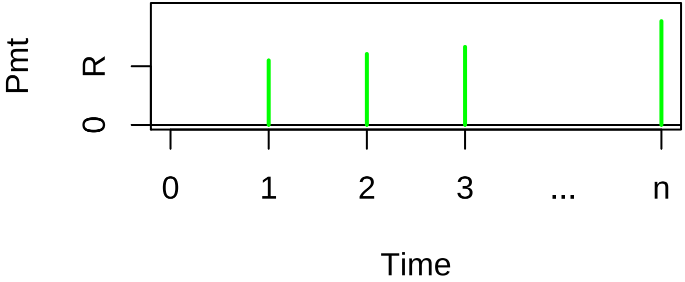
When examining a series of payments of R at the end of each of n periods, we previously found the present value of these payments to be

$$Ra_{\overline{n}|i} = R\left(\frac{1}{1+i}\right) + R\left(\frac{1}{1+i}\right)^2 + \cdots + R\left(\frac{1}{1+i}\right)^n.$$



The expression $Ra_{\overline{n}|i}$ uses the dollar growth rate i to view the values of each future payment. But this viewpoint does not take into account the buying power of each of those future payments. Suppose we desire each payment to have the buying power that R dollars have today.

In view of the inflation rate r, the respective payments would need to be: $R(1+r)$ at the end of year 1, $R(1+r)^2$ at the end of year 2, $\cdots$, and $R(1+r)^n$ at the end of year n.



Assuming a dollar growth rate (interest rate) of i, the present value (the price) of this sequence of payments with constant buying power of R in today's dollars is:

$$R\frac{(1+r)}{(1+i)} + R\frac{(1+r)^2}{(1+i)^2} + \cdots + R\frac{(1+r)^n}{(1+i)^n} = R(1+r)\frac{\left[1-\left(\frac{1+r}{1+i}\right)^n\right]}{(i-r)}$$

Another equivalent expression for this sum is:

because $(1+i') = \frac{(1+i)}{(1+r)}$. This present value of a sequence of payments with the constant buying power of R in today's dollars uses the net growth interest rate of i'. It is inflation adjusted and hence the price (present value) is larger than $Ra_{\overline{n}|i}$ because $i' < i$ (assuming $r > 0$).

The Federal Government now issues bonds called TIPS (Treasury Inflation Protection Securities). The coupon payments and the redemption value (face value) of these bonds are both indexed to the Consumer Price Index (CPI). As the CPI increases (decreases) so do the coupon payments and the face value.

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Example: You buy an annuity that will pay you \$25,000 in today's dollars at the end of each of the next 10 years. Suppose the annual effective interest rate is 8% and the rate of inflation is 1% during the first five years and 3% during the second five. What should you pay for this annuity?

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The present value of these payments is

$$P = 25,000 \left[\begin{array}{c} \left(\frac{1.01}{1.08}\right) + \cdots + \left(\frac{1.01}{1.08}\right)^5 + \left(\frac{1.01}{1.08}\right)^5 \left(\frac{1.03}{1.08}\right) + \\ \cdots + \left(\frac{1.01}{1.08}\right)^5 \left(\frac{1.03}{1.08}\right)^5 \end{array} \right]$$

During the first five years $i' =$

During the second five years $i' =$

We can also write our solution as

$$P = 25000 \left[a_{\overline{5}|.069307} + \left(\frac{1.01}{1.08} \right)^5 a_{\overline{5}|.048544} \right]$$

$$= 180,429.26 \quad \text{dollars.}$$

Exercise 9-3:

Money is invested for five years in a savings account earning 7% effective. If the rate of inflation is 10%, find the percentage of purchasing power lost during the period of investment.

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Exercise 9-5:

You deposit x in an account today in order to fund your retirement. You would like to receive 50K per year, in today's dollars, at the end of each year for a total of 12 years, with the first payment occurring seven years from now. The inflation rate will be 0.0% for the next 6 years and 1.2% per annum thereafter. The annual effective rate of return is 6.3%. Calculate x to the nearest dollar.

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