

SHOW FORMULAS USED, YOUR WORK AND REASONING ON ALL QUESTIONS

1. An installment loan is repaid with equal annual payments of \$400 at the end of each year for 10 years. This amortized loan has an effective annual interest rate of 5%.

(a) What is the outstanding loan balance **right after the 4th payment?** (7 pts)

$$B_4 = 400 a_{\overline{10-4}|.05} = 400 (5.075692067) \\ = \underline{\underline{2,030.28}}$$

(b) What amount is paid in interest on the 5th payment?

(5 pts)

$$B_4 (.05) = \underline{\underline{101.51}}$$

(c) What amount is paid toward principal on the 5th payment?

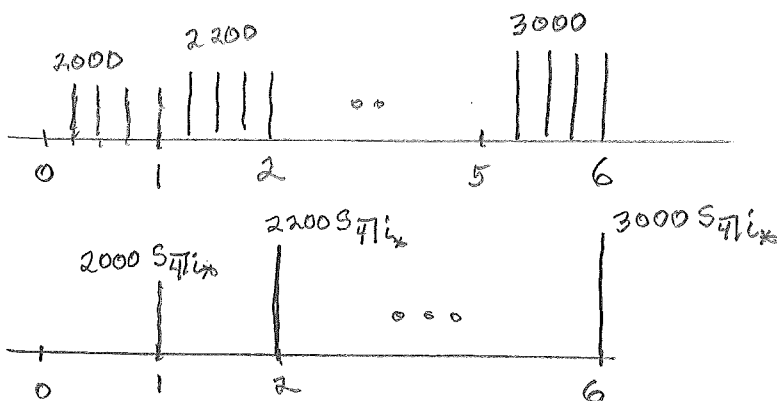
(4 pts)

$$400 - 101.51 = \underline{\underline{298.49}}$$

(d) [NOT RELATED TO WHAT IS ABOVE] A amortized loan of \$25,000 is repaid with equal annual payments over 12 years. If the annual effective interest rate is 4%, what is the annual payment amount? (7 pts)

$$\frac{25,000}{a_{\overline{12}|.04}} = \frac{25,000}{9.38507376} = \underline{\underline{2,663.80}}$$

2. Starting in year 2017, an annuity pays at the end of every quarter of the year for 6 years (24 total payments). Payments during the first year are \$2,000 each quarter. The four payments during year 2 are each \$2,200. The payments go up by \$200 for each subsequent year, but are level within any year. During year 6 the payments are therefore each \$3,000. If the annual effective interest rate is 5%, find the present value (as of January 1, 2017) of this annuity. (15 pts)



.05 eff. annual interest

$$(1+.05) = (1+i_*)^4$$

$$i_* = .01227223$$

$$s_{4|i_*} = 4.07423768$$

$$PV = 2000 s_{4|i_*} a_{\overline{6}|.05} + 200 s_{4|i_*} \left[\frac{a_{\overline{6}|.05} - 6 \left(\frac{1}{1.05} \right)^6}{.05} \right]$$

$$= 2000 (4.07423768) (5.07569206)$$

$$+ 200 (4.07423768) [11.96799375]$$

$$= 41,359.15 + 9,752.09$$

$$= \underline{\underline{51,111.24}}$$

3. A person borrows \$25,000 and repays the loan via the sinking fund method making level (constant) annual payments over 6 years. Interest is paid using an annual effective interest rate of 5%. Deposits are also required into a sinking fund that grows with an annual effective interest rate of 4%. Find the interest paid in each annual payment, the sinking fund deposit in each annual payment and the total of each payment (interest plus sinking fund deposit) for the loan to be paid in full at the end of the six years. (15 pts)

$$\begin{aligned}\text{Interest per Pmt} &= (.05)(25,000) \\ &= \underline{\underline{1,250}}\end{aligned}$$

$$\text{Dep. in Sinking Fund} = \frac{25,000}{s_{\overline{6}|.04}}$$

$$= \frac{25000}{6.63297546}$$

$$= \underline{\underline{3,769.05}}$$

$$\text{Total Pmt} = 1250 + 3,769.05$$

$$= \underline{\underline{5,019.05}}$$

4. An annuity pays level payments of \$500 at the beginning of every month for three years. The effective annual interest rate is 6%.

(a) What is the present value of this annuity at the time of the first payment?

$$(1.06) = (1 + i_*)^{12} \quad i_* = .00486755 \quad \begin{matrix} (15 \text{ pts}) \\ \text{eff. monthly} \\ \text{interest rate} \end{matrix}$$

$$PV = 500 \ddot{a}_{\overline{36}|i_*} = 500 (33.10933859) \\ = \underline{\underline{16,554.67}}$$

(b) What is the value of this annuity at the time $t = 3$ (years)?

(5 pts)

$$FV = \underset{\text{(above)}}{PV} (1.06)^3 = \underline{\underline{19,716.88}}$$

Move the vision point 3 years !

5. A 5-year annuity makes continuous payments with payment function

$$f(t) = \begin{cases} 100 & \text{for } 0 \leq t \leq 2 \\ 200 & \text{for } 2 < t \leq 5 \\ 0 & \text{elsewhere} \end{cases}$$

If there is a constant force of interest of $\delta = .05$, then find the present value of this continuous annuity. (12 pts)

$$\begin{aligned} PV &= \int_0^5 f(t) e^{-\delta t} dt \\ &= \int_0^2 100 e^{-.05t} dt + \int_2^5 200 e^{-.05t} dt \\ &= \frac{100}{(-.05)} e^{-.05t} \Big|_0^2 + \frac{200}{(-.05)} e^{-.05t} \Big|_2^5 \\ &= -2000 (e^{-.1} - 1) - 4000 (e^{-.25} - e^{-.1}) \\ &= 190.33 + 504.15 \\ &= \underline{\underline{694.48}} \end{aligned}$$

6. The following short answer questions are not related to one another.

(a) Suppose $s_{\overline{15}|i} = 20.55240852$. Find i . (Indicate your procedure.) (5 pts)

$$N = 15 \quad FV = 20.55240852 \quad PMT = -1$$

$$i = .0435$$

(b) Assume a nominal annual interest rate of 8% convertible quarterly. Find the effective 2-year interest rate. [SET THIS UP - DO NOT COMPUTE IT] (5 pts)

.02 eff. interest rate per quarter

i_x = eff. 2-year interest rate

$$(1 + i_x) = (1.02)^8$$

$$i_x = .171659$$

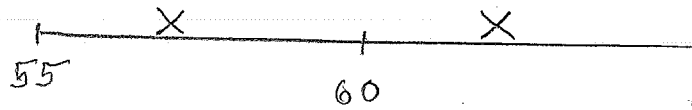
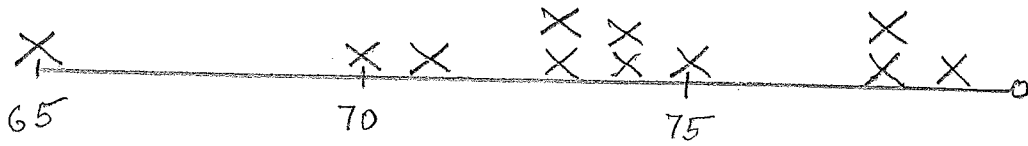
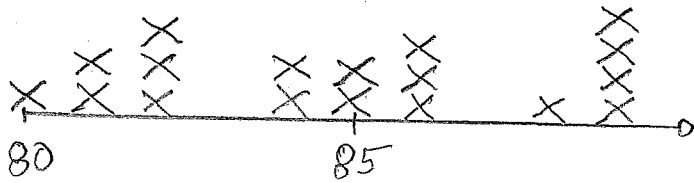
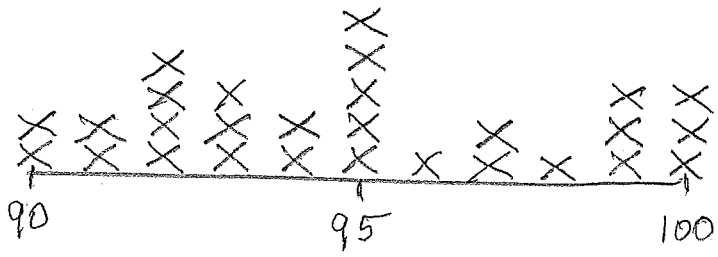
(c) Consider an investment that pays \$100 at the end of year 2 and an additional \$500 at the end of year 4. Suppose the interest rate throughout the first 2 years is 3%, for year 3 it is 4% and for year 4 it is 5%. What is the present value of this investment (combined payments) under the portfolio method? [SET THIS UP - DO NOT COMPUTE IT] (5 pts)

$$PV = 100 \left(\frac{1}{1.03} \right)^2 + 500 \left(\frac{1}{1.03} \right)^2 \left(\frac{1}{1.04} \right) \left(\frac{1}{1.05} \right)$$

STA 4183

EXAM 2

2016



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