Chapter 06 - Bonds and Other Securities

Section 6.2 - Bonds

Bond - an interest bearing security that promises to pay a stated amount of money at some future date(s).

maturity date - date of promised final payment

term - time between issue (beginning of bond) and maturity date

callable bond - may be redeemed early at the discretion of the borrower

putable bond - may be redeemed early at the discretion of the lender

redemption date - date at which bond is completely paid off - it may be prior to or equal to the maturity date

Bond Types:

Coupon bonds - borrower makes periodic payments (coupons) to lender until redemption at which time an additional redemption payment is also made

- no periodic payments, redemption payment includes original loan principal plus all accumulated interest

Convertible bonds - at a future date and under certain specified conditions the bond can be converted into common stock

Other Securities:

Preferred Stock - provides a fixed rate of return for an investment in the company. It provides ownership rather that indebtedness, but with restricted ownership privileges. It usually has no maturity date, but may be callable. The periodic payments are called dividends. Ranks below bonds but above common stock in security. Preferred stock is bought and sold at market price.

Common Stock - an ownership security without a fixed rate of return on the investment. Common stock dividends are paid only after interest has been paid on all indebtedness and on preferred stock. The dividend rate changes and is set by the Board of Directors. Common stock holders have true ownership and have voting rights for the Board of Directors, etc. The price of common stock is more volatile than that of preferred stock. Common stock is bought and sold at market price.

Example:

A 26-week T-Bill is purchased for $9,650 and matures at $10,000. What is its yield?

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T-Bills use actual/360 and discounting as their yield basis. Therefore,

$$5055.55x = 350$$

$$x = .06923.$$

Also, what is the annual effective yield rate assuming the investment is for exactly 1/2 year?

$$9650(1+x)^{1/2} = 10000$$

$$(1+x)^{1/2} = 1.03626943$$

$$1+x = 1.07385$$

$$x = .07385.$$

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Section 6.3 - The Price of a Bond

Set the price (value today) of a bond to be the present value of all future payments upon issue of the bond or right after a coupon payment. We assume all obligations are paid and the bond continues to maturity.

Notation:

$P =$ price of the bond

$F =$ face value (par value) of the bond, often (but not always) the amount paid at maturity.

$C =$

(when C=F it is called a par value bond).

$r =$ coupon rate (typically this is a semiannual rate.)

$Fr =$

$g =$ modified coupon rate in terms of the redemption value

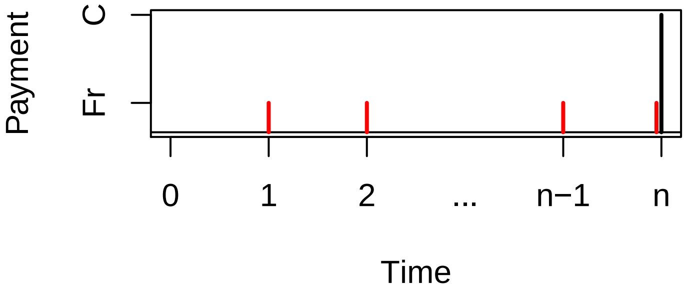
$Cg = Fr$, so $g = r$ whenever $C = F$

$i =$ yield rate, i.e. interest rate earned if bond is held to maturity

$n =$ number of coupon payment periods current date to redemption (maturity)

$K =$ present value (given i) of the redemption (maturity) payment, i.e. $K = C\nu^n$

$G =$ base amount of bond, defined by $Gi = Fr$.



Determining The Price of a Bond

The Basic Formula

$$= \quad Fr\left(\frac{1-(\frac{1}{1+i})^n}{i}\right)+C\left(\frac{1}{1+i}\right)^n.$$

Premium / Discount Formula

$$P = Fra_{\overline{n}|} + C\nu^n$$

$$= Fra_{\overline{n}|} + C(1 - ia_{\overline{n}|})$$

$$= C + (Fr - Ci)a_{\overline{n}|} \qquad \text{or}$$

Base Amount Formula

$$P = Fra_{\overline{n}|} + C\nu^n$$

$$= Gia_{\overline{n}|} + C\nu^n$$

$$= G(1 - \nu^n) + C\nu^n \qquad \text{or}$$

Makeham's Formula

$$P = Fra_{\overline{n}|} + C\nu^n$$

$$= C\nu^n + Cg\left(\frac{1-\nu^n}{i}\right)$$

Example:

Two \$1000 bonds redeemable at par at the end of the same period of time are bought to yield 4% convertible semiannually. One bond costs \$1136.76 and has a coupon rate of 5% payable semiannually. The other has a coupon rate of 2.5% payable semiannually. Find the price of the second bond.

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Applying the basic formula to the first bond produces

$$250(1.02)^{-n} = 113.24$$

$$n = \frac{-ln(.45296)}{ln(1.02)} = 40.$$

Now applying the basic formula to the second bond, produces

$$P = 1000(.0125)\frac{[1-(1.02)^{-40}]}{.02} + 1000(1.02)^{-40}$$

$$= \$794.83.$$

Exercise 6-2:

A 10-year accumulation bond with an initial par value of $1000 earns interest of 8% compounded semiannually. Find the price to yield an investor 10% effective.

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Exercise 6-7:

A $1000 par value bond maturing at par with $100 annual coupons is purchsed for $1110. If K=$450, find the base amount.

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Exercise 6-8:

An investor owns a $1000 par value 10% bond with semiannual coupons. The bond will mature at the end of 10 years. The investor decides an 8 year bond would be preferable. Current yield rates are 7% convertible semiannually. The investor uses the proceeds from the sale of the 10% bond to purchase a 6% bond with semiannual coupons, maturing at par at the end of 8 years. Find the par value of the 8-year bond.

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Section 6.4 - Premium and Discount

One of the most important formulas for the price of a bond is the premium/discount formula:

$$P = C + C(g - i)a_{\overline{n}|}.$$

Sometimes $P > C$, i.e. the price of the bond is greater than its redemption value. When this occurs, the bond is said to sell at premium. In this case, $g > i$, i.e. the coupon rate exceeds the effective yield rate of the bond. Each coupon payment includes more than the designated interest. It also returns some of the principal of the investment.

The book value, B_t, of the bond immediately after the t^{th} coupon has been paid is:

$$B_t = Fra_{\overline{n-t}|i} + C\left(\frac{1}{1+i}\right)^{n-t} \qquad \text{(PV of future payments)}$$

$$=$$

Note that $B_0 = P$ and as t increases the values decrease (when $g > i$) to $B_n = C$.

We can view the t^{th} coupon payment of $Fr = Cg$ as part interest:

$$\mathrm{I}_t \equiv iB_{t-1} = iC + iC(g-i)a_{\overline{n-t+1}|}$$

$$= iC[1 + (g-i)a_{\overline{n-t+1}|}]$$

and part principal adjustment (principal replacement):

$$
\begin{aligned}
P_t &= Fr - \mathrm{I}_t \\
&= Cg - iC - iC(g-i)a_{\overline{n-t+1}|} \\
&= C(g-i)\left[1 - ia_{\overline{n-t+1}|}\right] \\
&= C(g-i)\nu^{n-t+1}.
\end{aligned}
$$

Note also that

$$
\begin{aligned}
B_t + P_t &= C + C(g-i)a_{\overline{n-t}|} + C(g-i)\nu^{n-t+1} \\
&= C + C(g-i)\left[a_{\overline{n-t}|} + \nu^{n-t+1}\right] \\
&= C + C(g-i)a_{\overline{n-t+1}|} \\
&= B_{t-1},
\end{aligned}
$$

i.e. these P_t values are replacing the principal.

The values I_t, P_t and B_t can form an amortization schedule for the bond.

Example:

Consider a \$1000 two-year bond with 7% coupons paid semiannually and a yield of 5% convertible semiannually. Here

$$C = F = 1000 \qquad \text{and} \qquad r = g = .035$$

$$i = .025 \qquad \text{and} \qquad n = 4.$$

$$Fr = Cg = 35.$$

The price of the bond is:

$$= 1037.62 \equiv B_0 \quad > \quad C = 1000$$

The amortization schedule uses $Ci = 25$ and becomes

t	Cg Coupon	I_t Interest	P_t Prin Adj	B_t Book Value
0				1037.62
1	35.00	25.94	9.06	1028.56
2	35.00	25.71	9.29	1019.27
3	35.00	25.48	9.52	1009.75
4	35.00	25.25	9.75	1000.00
	140.00	102.38	37.63	

In general,

Total Coupons $=$

Total Principal Adjustment $= P - C$

Total Interest $=$

When $P < C$, that is, when $g < i$, the bond is said to be sold at discount. The values of P_t are now negative, so $-P_t$ is designated as the discount amount in the t^{th} coupon and $-P_t$ is displayed in the amortization schedule.

In this setting the coupon payments are less than what is needed to cover the interest owed to the investor to achieve the yield rate i. This unpaid interest is accumulated to the maturity date of the bond and more is returned then, C, than what was originally invested, P.

The book value increases over time from $B_0 = P$ at time $t = 0$ to $B_n = C$ at time $t = n$.

When $P = C$, then $g = i$, and the bond is said to be

In this case, the coupon rate and the yield rate are equal. Moreover, the coupon payments contain only interest on the original amount that is invested in the bond.

Exercise 6-13:

A 10-year bond with semiannual coupons is bought at a discount to yield 9% convertible semiannually. If the amount for accumulation of discount in the next-to-last coupon is $8, find the total amount of accumulation of discount during the four years in the bond amortization schedule.

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Section 6.5 - Valuation of a Bond Between Coupon Dates

The book value B_t at a time t immediately after a coupon payment represents the value (price) of the bond that achieves the designated yield, i, over the remainder of the of the bond's term until maturity. Note that

$$B_{t+1} = Fra_{\overline{n-t-1|}} + C\nu^{n-t-1}$$

$$= Fr\left(\frac{1-\nu^{n-t-1}}{i}\right) + C\nu^{n-t-1}$$

$$= Fr\left(\frac{(1+i)-\nu^{n-t-1}}{i}\right) + C\nu^{n-t-1} - Fr$$

$$= (1+i)\left(Fr\frac{(1-\nu^{n-t})}{i} + C\nu^{n-t}\right) - Fr$$

$$=$$

So the book value right after the $(t+1)^{th}$ coupon payment, is the accrued book value from time t, minus the coupon payment at $t+1$.

What is an appropriate price of the bond at time $t+k$, where $0 < k < 1$? Define

$$Fr_k = \text{accrued coupon value at } t+k$$

$$B^f_{t+k} = \text{flat price of the bond at time } t+k$$

(the money that would change hands in its sale)

and

$$B^m_{t+k} = \text{market price of the bond at time } t+k \ .$$

Here

To determine an appropriate market price, we therefore must find values for both B^f_{t+k} and Fr_k. There are several methods for determining these values.

To determine an appropriate market price, we therefore must find values for both B^f_{t+k} and Fr_k. There are several methods for determining these values.

(1) Theoretical Method (compound interest for both)

$$Fr_k = Fr\left(\frac{(1+i)^k - 1}{i}\right)$$

$$B^f_{t+k} = B_t(1+i)^k$$

(2) Practical Method (simple interest for both)

$$Fr_k =$$

$$B^f_{t+k} =$$

(3) Semi-Theoretical Method

$$Fr_k = kFr$$

$$B^f_{t+k} = B_t(1+i)^k$$

In all three cases,

$$Fr_0 = 0 \qquad B^f_{t+0} = B^m_{t+0} = B_t$$

$$Fr_1 = Fr \qquad B^f_{t+1} = B_t(1+i) \qquad \text{and}$$

$$B^m_{t+1} = B_t(1+i) - Fr = B_{t+1}.$$

Example:

Using the example setting of section 6.4 and all three methods, find the above values at $t = 2 + 1/3$, i.e. two months after the payment of the 2^{nd} coupon.

$$(1)\ Fr_{1/3} = 35\left(\frac{(1.025)^{1/3} - 1}{.025}\right) = 11.57$$

$$B^f_{2+1/3} = 1019.27(1.025)^{1/3} = 1027.69$$

$$B^m_{2+1/3} = 1027.69 - 11.57 = 1016.12$$

(2) $Fr_{1/3} =$

$B^f_{2+1/3} =$

$B^m_{2+1/3} = 1027.76 - 11.67 = 1016.09$

(3) $Fr_{1/3} = 11.67$

$B^f_{2+1/3} = 1027.69$

$B^m_{2+1/3} = 1027.69 - 11.67 = 1016.02$

Section 6.6 - Determining the Yield Rate of a Bond

Recall the premium discount formula

$$P = C + C(g - i)a_{\overline{n}|i}.$$

We see from this expression that even if we know P, C, and g, it will be difficult to solve for i. So we first note that

$$\frac{1}{a_{\overline{n}|i}} = \frac{i}{1-(1+i)^{-n}}$$

$$= \frac{i(1+i)^n}{(1+i)^n - 1}$$

$$= \frac{\sum_{j=0}^{n} i \binom{n}{j} i^j}{\sum_{j'=0}^{n} \binom{n}{j'} i^{j'}}$$

long division of polynomials then produces

$$= \frac{1}{n}\left[1 + \left(\frac{n+1}{2}\right) i + \left(\frac{n^2-1}{12}\right) i^2 + \cdots\right]$$

The quantity $1/a_{\overline{n}|i}$ is approximated by the first two terms, i.e. $\frac{1}{n}\left[1+\left(\frac{n+1}{2}\right)i\right]$. Now define

$$k \equiv \frac{P-C}{C} = (g-i)a_{\overline{n}|i}$$

Then

$$i = g - \frac{k}{a_{\overline{n}|i}} \doteq g - \frac{k}{n}\left(1+\frac{n+1}{2}i\right).$$

This is an expression that we can then solve for i, producing

as a quick approximate solution for the yield rate of a bond in terms of g, n, P and C.

An exact solution can be found with a financial calculator.

Example:

What is the yield for a \$1000 par value 10 year bond selling for \$1100 which pays semiannual coupons with an 8% coupon rate convertible semiannually?

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The approximation produces

$$k = \frac{1100 - 1000}{1000} = .1 \qquad \text{and}$$

$$i \doteq \frac{.04 - (\frac{.1}{20})}{1 + (\frac{21}{40})(.1)} = .033254$$

and an annual yield of $\doteq$ .066508. The exact solution from a financial calculator is:

$$i = .0330852$$

or an annual yield of .066170.

Exercise 6-22:

A \$100 bond with annual coupons is redeemable at par at the end of 15 years. At a purchase price of \$92, the yield rate is exactly 1% more than the coupon rate. Find the yield rate.

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Exercise 6-23:

A n-year par value bond with 4.2% annual coupons is purchased at a price to yield an annual effective rate i. You are given:

(a) If the annual coupon rate had been 5.25% instead of 4.2%, the price would have increased by \$100.

(b) At the time of purchase, the present value of all the coupon payments is equal to the present value of the bond redemption value of \$1000.

Find i.

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Section 6.7 - Callable and Putable Bonds

A callable bond is one for which the issuer (borrower) has the option to redeem the bond prior to the maturity date (usually at one of a list of specified possible call dates). The redemption value C may possibly differ from the face value of the bond F. If so, these potential redemption values should be specified in advance in the bond description.

In a callable bond the potential call (or maturity) of the bond is beyond the control of the bond purchaser. So when computing the yield (at a fixed price) or the price (at a fixed yield) we should assume a worst case scenario, i.e. that the borrower uses the call date that disadvantages us the most. By doing this we will ensure that the actual yield is no worse (and may be better than) the specified yield i.

For instance, if $C = F$ at all call dates and

(a) $P > C$ (the bond is purchased at premium) then the earliest call date is the most disadvantageous for the purchaser.

(b) $P < C$ (the bond is purchased at discount) then the maturity date is the most disadvantageous for the purchaser.

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When the redemption value depends on the call date, we must assess each call date separately to determine which call date is the least advantageous to the purchaser.

Example: A $1000 par value bond pays coupons semiannually at a rate of 5% convertible semiannually over the 6 year term of the bond. The bond is callable at $1080 after 2 years and $1040 after 4 years. What price should a buyer pay to ensure a 6% yield convertible semiannually?

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Coupon payments are \$25 semiannually. If it is called in 2 years,

$$P =$$

$$= 1080 + (25 - 32.4)\left(\frac{1-(\frac{1}{1.03})^4}{.03}\right) = 1052.49.$$

If called at 4 years, the price should be

$$P = 1040 + (25 - 1040(.03))a_{\overline{8}|.03}$$

$$= 1040 + (25 - 31.2)\left(\frac{1-(\frac{1}{1.03})^8}{.03}\right) = 996.48.$$

Finally, if held to maturity at 6 years, the price should be

$$P = 1000 + (25 - 1000(.03))a_{\overline{12}|.03}$$

$$= 1000 + (25 - 30)\left(\frac{1-(\frac{1}{1.03})^{12}}{.03}\right) = 950.23.$$

So to ensure a yield of 6% convertible semiannually, the price offered should be \$950.23.

Exercise 6-7:

A $1000 par value bond with coupon payments semiannually at 9% was called for $1100 prior to maturity. The bond was bought for $918 immediately after a coupon payment and was held to call. the nominal yield rate convertible semiannually was 10%. Calculate the number of years the bond was held.

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Section 6.10 - Other Securities

Both preferred stock and perpetual bonds provide payments of Fr in perpetuity. So when i is the effective interest rate per payment period, the price satisfies

Example:

An investment of $1000 in a preferred stock that pays $20 semiannually has what yield?

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$$i = \frac{Fr}{P} = \frac{20}{1000} = .02 \qquad \text{that is,}$$

$$2(.02) = .04 \qquad \text{or 4\% annual yield convertible semiannually.}$$

Common Stock

What price should be paid for common stock? One answer is the present value of future dividends. This requires assumptions about the pattern of future dividend payments. Of course, predictions of future dividends are problematic.

Example:

A stock has just paid a quarterly dividend of \$0.25 per share. It's dividends are assumed to grow perpetually at a nominal annual rate of 2% per year convertible quarterly. What price per share would produce a 5% nominal annual yield convertible quarterly?

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If i denotes the effective quarterly interest rate and j the effective quarterly dividend growth rate, it follows that

$$i = .05/4 = .0125 \qquad \text{and} \qquad j = .02/4 = .005.$$

The price should be the present value of all future payments, that is

Using the SGS, we get

$$P = Fr\left(\frac{1+j}{1+i}\right)\lim_{n\to\infty}\left[\frac{1-\left(\frac{1+j}{1+i}\right)^n}{1-\left(\frac{1+j}{1+i}\right)}\right]$$

$$= Fr\left(\frac{1+j}{1+i}\right)\left(\frac{1+i}{i-j}\right) \qquad \text{as long as } 0 < j < i$$

$$= Fr\left(\frac{1+j}{i-j}\right).$$

Putting in the numbers produces

$$P = (.25)\left(\frac{1+.005}{.0125-.005}\right) = \$33.50.$$

Exercise 6-38:

A $100 par value 10% preferred stock with quarterly dividends is bought to yield 8% convertible quarterly into perpetuity. However, the preferred stock is actually called at the end of 10 years at par. Find the nominal yield rate convertible quarterly that an investor would actually earn over the 10-year period.

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