Interest Rate Conversions

The Interest Rate Factor

When i is the effective rate of interest for some interest period,

$$i = \frac{(\text{end balance}) - (\text{beg. balance})}{(\text{beg. balance})}.$$

Solving this for the (end balance) produces

$$\boxed{(\text{beg. balance})(1+i) = (\text{end balance})}$$

In this form, we call

$$(1+i)$$

the interest rate factor for that period.

In fact we identify the effective interest rate through this factor. That is, if

$$(\text{beg. balance})\,x = (\text{end balance}),$$

the effective interest rate i is equal to

$$i = x - 1 \qquad \text{because} \qquad x = (1+i).$$

Note also that the discount factor satisfies

$$(\text{beg. balance}) = \frac{1}{(1+i)}(\text{end balance})$$

i.e. $\quad V = \dfrac{1}{(1+i)}$ can also be used to identify i, the effective interest rate for the interest period.

Nominal Interest Rates

Suppose $i^{(4)}$ is the nominal annual interest rate convertible quarterly (4 times per year). Then the annual interest rate factor is

$$(1+i) = \left(1 + \frac{i^{(4)}}{4}\right)^4$$

where i is the effective interest rate per year. Note that this factor is a product of 4 terms

$$\left(1+\frac{i^{(4)}}{4}\right)\left(1+\frac{i^{(4)}}{4}\right)\left(1+\frac{i^{(4)}}{4}\right)\left(1+\frac{i^{(4)}}{4}\right)$$

with each term describing the growth

in a single quarter. Thus

$$\frac{i^{(4)}}{4}$$

is the <u>effective rate of interest in each quarter</u>.

For general m, the yearly interest rate factor is

$$(1+i) = \left(1+\frac{i^{(m)}}{m}\right)^m = \underbrace{\left(1+\frac{i^{(m)}}{m}\right)\left(1+\frac{i^{(m)}}{m}\right)\cdots\left(1+\frac{i^{(m)}}{m}\right)}_{m \text{ factors}}$$

Here

i is the effective annual interest rate

and

$\frac{i^{(m)}}{m}$ is the effective m^{th}ly interest rate

because

$\left(1+\frac{i^{(m)}}{m}\right)$ is the factor that describes

the growth is each m^{th}ly sub period.

Use of Interest Rates in Annuities (and Other Settings

In an annuity,

$a_{\overline{n}|i}$

↑ uses i = effect interest rate in a payment period.

Thus to use the formula for an annuity immediate, we must find the effective interest rate for the gap between payments.

So in annuity problem settings (and others) we often must find (solve for) the effective interest rate in a payment period before we can proceed to answer the main question. We identify the effective interest rate by finding the appropriate interest rate factor.

Knowing i for the payment period you can always find d for the payment period with $d = \frac{i}{1+i}$

Example 1 3% is an effective annual interest rate, what is the effective five year interest rate?

$$(1+.03)^5 = (1+i) \quad \text{or} \quad i = .15927407$$

Example 2 8% is a nominal annual interest rate compounded quarterly. Find the effective monthly interest rate.

$$\left(1+\frac{.08}{4}\right)^4 = (1+i) = (1+i_o)^{12}$$

eff. annual effective monthly

$$i_o = .0066227095 6$$

Example 3 6% is the annual interest rate, find the effective daily rate.

$$(1+.06) = (1+i_o)^{365}$$

effective daily

$$i_o = .0001596535 9$$

Exercise A   8% is a nominal annual interest rate convertible monthy. Find an effective 3-year interest rate

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Exercise B   15% is an effective 2-year interest rate. Find the corresponding effective quarterly interest rate.

Exercise C   5% annual discount rate. What is the effective monthy interest rate?

Ans: A .27023705177   B .01762374

C .00428358965