STA 4183 First Exam - Fall 2016 PRINT Name: Key

UFID: ____________

SHOW FORMULAS USED, YOUR WORK AND REASONING ON ALL QUESTIONS

1. The accumulation function of an account is $a(t) = 1+(.01)t+(.005)t^3$, for $0 \leq t \leq 5$.

(a) Find the effective rate of interest during year 2, ie between $t = 1$ and $t = 2$.

(7 pts)

$$\frac{a(2)-a(1)}{a(1)} = \frac{\cancel{1}+(.01)2+(.005)8-\cancel{1}-(.01)-.005}{1+.01+.005}$$

$$= \frac{.045}{1.015} = .044335$$

(b) Find the effective rate of discount during year 4, ie between $t = 3$ and $t = 4$.

(7 pts)

$$\frac{a(4)-a(3)}{a(4)} = \frac{1+(.01)4+(.005)64-1-(.01)3-(.005)27}{1+(.01)4+(.005)64}$$

$$= \frac{.195}{1.36} = .14338235$$

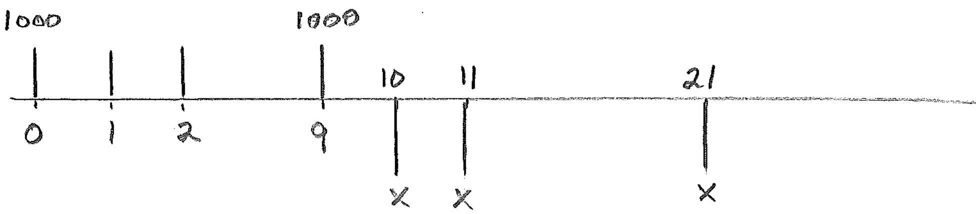
(c) What is the force of interest at time $t = 2$? (7 pts)

$$a'(t) = (.01) + (.015)t^2$$

$$\delta_2 = \frac{a'(2)}{a(2)} = \frac{(.01)+(.015)4}{1+(.01)2+(.005)8} = \frac{.07}{1.06}$$

$$= .0660377$$

2. A deposit of \$1,000 is made at the beginning of each year into an account for 10 years (last deposit at $t = 9$). Beginning at the end of the 10th year ($t = 10$), equal annual payments of X are withdrawn from the account. The account is exhausted (balance of zero) with the 12th payment of X at time $t = 21$. The account has an annual effective interest rate of 5%. Find X. (15 pts)



$$1000\,\ddot{s}_{\overline{10}|.05} = X\,\ddot{a}_{\overline{12}|.05}$$

$$X = \frac{1000\,\ddot{s}_{\overline{10}|.05}}{\ddot{a}_{\overline{12}|.05}}$$

$$= \frac{1000\left[\dfrac{(1.05)^{10}-1}{(.05)/(1.05)}\right]}{\left[\dfrac{1-\left(\frac{1}{1.05}\right)^{12}}{\left(\frac{.05}{1.05}\right)}\right]}$$

$$= \frac{1000(13.206787)}{(9.30641422)} = \$1,419.11$$

3. Alice deposits \$400 into an account earning simple interest at 6% per year. Jerry deposits \$500 into an account earning simple interest at 2% per year. How long will it take for the balance in Alice's account to be twice as large as the balance in Jerry's account? (15 pts)

$$A_A(t) = 400(1+(.06)t)$$

$$A_J(t) = 500(1+(.02)t)$$

Find t for which

$$400(1+(.06)t) = (2)(500)(1+(.02)t)$$

$$(24-20)t = 600$$

$$t = \frac{600}{4} = 150 \text{ years}$$

4. An investment has a force of interest function of

$$\delta_t = (0.15) + (0.10)t \quad \text{for } 0 \le t$$

(a) Find the accumulation function. (7 pts)

$$\int_0^t [(.15) + (.1)r]\,dr = (.15)r + (.05)r^2 \Big|_0^t$$

$$= .15t + (.05)t^2$$

$$a(t) = e^{(.15t + (.05)t^2)}$$

(b) If this investment accumulates to \$400 at $t = 2$, find its value at $t = 4$. (7 pts)

$$A(4) = 400\left[\frac{a(4)}{a(2)}\right]$$

$$= \frac{400\, e^{.6+.8}}{e^{.3+.2}} = \frac{400\,(4.05519997)}{(1.6487213)}$$

$$= 983.84$$

(c) Interpret (in words) the meaning of the force of interest function value at $t = 1$.

$\delta_1 = .25$ (3 pts)

It is the rate of change in $A(t)$ at $t=1$ relative to the 4 size of $A(t)$ at $t=1$.

5. A loan of \$25,000 is repaid at the end of 5 years with a payment of \$32,000. Assume compound interest (discount) as applicable.

(a) Find the annual effective rate of interest of this loan. (7 pts)

$$(25{,}000)(1+i)^5 = 32{,}000$$
$$(1+i)^5 = 1.28$$
$$1+i = 1.0506111$$
$$i = .0506111$$

(b) Find the nominal annual rate of interest of this loan, if the interest is compounded quarterly. (7 pts)

$$(25{,}000)\left(1+\frac{i^{(4)}}{4}\right)^{20} = 32{,}000$$
$$\left(1+\frac{i^{(4)}}{4}\right)^{20} = 1.28$$
$$\left(1+\frac{i^{(4)}}{4}\right) = 1.01241949$$
$$i^{(4)} = .04967797$$

(c) NOT RELATED TO THE ABOVE SETTING – If the nominal annual rate of interest is 7% convertible semiannually, find the corresponding equivalent nominal annual rate of discount convertible quarterly. (5 pts)

$$\left(1+\frac{.07}{2}\right)^2 = \left(1-\frac{d^{(4)}}{4}\right)^{-4}$$
$$1-\frac{d^{(4)}}{4} = \left(1+\frac{.07}{2}\right)^{-1/2} = .98294.6374$$
$$d^{(4)} = .0682145$$

6. A stream of payments consists of \$500 at the end of year 1 ($t = 1$), \$750 at 1.5 years ($t = 1.5$), and \$1,000 at 3 years ($t = 3$). Assume a nominal annual rate of interest of 8% convertible semiannually.

(a) Find the present value of this stream of payments, ie its value at $t = 0$. (8 pts)

$$PV = 500\left(\frac{1}{1+\frac{.08}{2}}\right)^2 + 750\left(\frac{1}{1.04}\right)^3 + 1000\left(\frac{1}{1.04}\right)^6$$

$$= 462.278106 + 666.747269 + 790.314526$$

$$= 1{,}919.3399$$

(b) Find the value of this stream (all the payments) at $t = 2$. (5 pts)

$$V(2) = PV\,(1.04)^4$$

$$= 2{,}245.3562$$

Just move the vision point!

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Exam 1

Fall 2016