Chapter 5
Sections 5.1.1 and 5.1.2: Discrete Random Variables

Random Variables

- A quantity that (prior to observation) can be thought of as dependent on chance phenomena.
- Can be mapped to the real line
- Represented by Capital Letters. Write RV: X

Random Variables can be:

Discrete—can take on a countable number of possible outcomes

Continuous–takes on values on a continuous interval

Probability distribution (or probability mass functions or pmf) for discrete random variables is a list of all possible values of the variable x and the probability p(x) associated with each possible value. In addition,

1. $f(x_i)$ must sum to 1
2. $f(x_i)$ must be between 0 and 1

Ex. A certain type of solar yard light comes in a package of two. Let x denote the number of lights that work satisfactorily.

Number of Lights	Frequency	f(x)
0	1	
1	10	
2	89	

a.) $P(X=1)$

b.) $P(X>1)$

c.) $P(0<X<1) =$

d.) $f(0)=$

e.) $f(4) =$

Histogram of pmf

Cumulative Distribution Function (cdf)

F(x) = P(X= x) = ? f(x) for X a discrete RV

Number of Lights	Frequency	f(x)	F(x)
0	1	0.01	
1	10	0.10	
2	89	0.89	

Histogram

Use the cdf to answer the following questions.

a.) Find f(2).

b.) Find f(4) =

c.) Find F(-1) =

Section 5.1.3 Finding the mean and variance of discrete random variables.

The mean or the expected value of a discrete random variable of X is

$$m = E(X) =$$

The variance of a discrete random variable is

$$s^2 = Var(x) = \sum_{all\_x} (x_i - m)^2 f(x_i)$$

Examples

1.) Show that $s^2 = Var(x) = \sum_{all\_x} (x_i - m)^2 f(x_i) == \sum_{all\_x} x_i^2 f(x_i) - m^2$

2.) Use the light example, to answer the following questions.

Number of Lights	Frequency	f(x)
0	1	0.01
1	10	0.10
2	89	0.89

a. Find E(x).

b. Find s^2.

c. Find s .

3.) Suppose that there is a very simple auto insurance company. If you have an accident, you get $10,000. If you do not have an accident, you get $0. You have to pay $350 dollars in premiums. The insurance company has an actuarial table that gives the probability of a person with your age, sex, previous accident record and martial status being in an accident. Let's suppose that the probability that you are in an accident is 0.001. Find the expected gains ($) for the insurance company (per person, per year).

event	X	p(x)
accident		
no accident		

4.) Let W be the number of dots on the side of a ten sided dice.

a.) Describe the pmf. (Note: Sometimes probability distributions can be given as a formula.)

f(w) =

b.) Find EW.

c.) Var(w) =

d.) s =

e.)Write f(3) in terms of F(x).

Section 5.1.4: Binomial Distribution and Geometric Distribution

Terminology

$n! = n * n-1 * n-2 * ....3 * 2 * 1$

$\begin{pmatrix} n \\ x \end{pmatrix} = \left(\frac{n!}{x! * (n-x)!} \right)$ (read as n chose x)

Practice:

a. 5! =

b. $\begin{pmatrix} 5 \\ 3 \end{pmatrix} =$

Conditions of a Binomial Experiment

1. Experiment Consists of n Identical Trials
2. Outcome of each trial can be success/failure
3. Probability of a success is the same for each trail p
4. Trails are independent of each other.

If these conditions are true and X= the number of successes, then X has a Binomial Distribution with parameters n and p.

Ex. Determine if the following scenarios are examples of Binomial Experiments.

A. A coin is flipped 10 times and the number of heads are observed.

b. Ask 100 randomly selected students if they drank (or drink) when they were (are) underage. Make sure that the questioning is handled in a way that the respondent's answer would be confidential. Count the number of students who answer yes to the question.

c. Select a SRS of 10 switches from a shipment of 10,000 of them. Count the number of switches in the sample that do not conform to specifications. Suppose that unknown to us, 1,000 of the switches are nonconforming.

General Rule: Rule of thumb for Independence— Suppose each trail of an experiment can result in S or F and the sampling method is SRS, but the sampling is without replacement from a population of size N. If the sample size (the number of trials) n is at most 10% of the population size and p is not close to 1 or 0, the experiment can be analyzed as approximately a binomial experiment(n, p).

Formula for the Binomial

Ex. A new machine has 3 parts. Each one has a .2 of working. Each of these parts is independent of the other parts. Let X= number of parts that are working.
What is the distribution of X?

Sample Space	X	Probability

<u>pdf of the Binomial</u>

$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$ for x=0,1, ... n

= 0   otherwise

<u>Mean and Variance of the Binomial</u>

E(x) = np and the Var(x) = npq where q= 1-p

Example: A machine separates out all non-conforming parts. Non-conforming parts are classified as either “scrap” and rework”. It is known that 20% of nonconforming parts are reworkable. Suppose we inspect the next 10 nonconforming parts. U= # of reworkables in sample size of 10

a.) What is the distribution of U? ______________________________

b.) What is the probability that two are reworkable?

c.) What is the probability that at least two are reworkable?

d.) What is the mean?

e.) What is the variance?

Cumulative Distribution Function

Binomial with n = 10 and p = 0.200000

x	P(X <= x)
0.00	0.1074
1.00	0.3758
2.00	0.6778
3.00	0.8791
4.00	0.9672
5.00	0.9936
6.00	0.9991
7.00	0.9999
8.00	1.0000
9.00	1.0000
10.00	1.0000

Probability Density Function

Binomial with n = 10 and p = 0.200000

x	P(X = x)
0.00	0.1074
1.00	0.2684
2.00	0.3020
3.00	0.2013
4.00	0.0881
5.00	0.0264
6.00	0.0055
7.00	0.0008
8.00	0.0001
9.00	0.0000
10.00	0.0000

f.) What is the probability that four are reworkable? (Use Computer output)

g.) What is the probability that four or more are reworkable? (Use Computer output)

Geometric Distribution (p)

- X= is the order of the first success in a set of binomial trials.
- You sample until you get a success.

<u>pdf for the Geometric Distribution</u>

$f(x) = p(1-p)^{x-1}$ for x=1, 2, 3

0 otherwise

<u>cdf for the Geometric Distribution</u>

$1- F(x) = (1-p)^x$

<u>mean and variance of geometric distribution</u>

$$\text{mean} = E(x) = \frac{1}{p}$$

$$\text{variance} = Var(x) = \frac{1-p}{p^2}$$

Ex. Some biology students were checking the eye color for a large number of fruit flies. For the individual fly, suppose that the probability of white eyes is ¼ and the probability of red eyes is 3/4 , and that we may treat these observations as outcomes of a Binomial experiment. Answer the following questions. Let X= the number of trials needed until you count the first fly with white eyes.

a.)What is the pdf?

b.) Calculate the mean and variance.

c.) Find the probability that she has to look at four flies before she sees one with white eyes.

d.) Find the probability that she has to look at less than or equal to four flies before she sees one with white eyes.

e.)What is the probability that she has to look at least 5 flies are tested without finding one with white eyes?

Section 5.1.5 Poisson

Situation—When you are counting the number of times particular events occur in a given time or on a given physical object

Poisson Process

- The number of events have to be independent
- The probability of one event occurring in a short length of time h is approximately ?h
- The probability of two or more events occurring in a sufficiently short interval is essentially zero

X=# of occurrences of a "rare" event in a time (or space) interval

$\lambda =$ rate of occurrence

pdf of the Poisson distribution

$$f(x) = \frac{e^{-l}\, l^{x}}{x!} \text{ when x =0,1,2,3}$$

$$= 0 \quad \text{otherwise}$$

Mean and Variance of the Poisson

E(X) ? and **Var(X)** ?

Ex. A previous study revealed that on M through W, an average of 100 students walked into the library between 12:00 and 12:10 pm. Let X=# of students that enter the library. Assume X~Poisson.

a.)What is the rate of arrival per minute?

b.) What is the probability that between 10 and 12 students enter the library next Tuesday between 12:05 and 12:06 pm?

c.) What is E(X) and Var(X)?

d.)What would be the probability that between 8 and 16 students enter the library next Tuesday between 12:05 and 12:06 pm? (Use the computer.)

Cumulative Distribution Function

Poisson with mu = 10.0000

x	P(X <= x)
0.00	0.0000
1.00	0.0005
2.00	0.0028
3.00	0.0103
4.00	0.0293
5.00	0.0671
6.00	0.1301
7.00	0.2202
8.00	0.3328
9.00	0.4579
10.00	0.5830
11.00	0.6968
12.00	0.7916
13.00	0.8645
14.00	0.9165
15.00	0.9513
16.00	0.9730
17.00	0.9857
18.00	0.9928
19.00	0.9965
20.00	0.9984