

HW 1 for Stat 7249 - Spring 2009

Due January 19

Reading in text for this assignment

- Chapters 3-5

Datasets

- none for this assignment

1. Problem 3.8 in DB
2. Cumulants and Cumulant generating function. Let $M_y(t)$ be the moment generating function for a random variable Y (we will assume it is finite for t in a neighborhood of 0). The cumulant generating function is defined as $K_y(t) = \log M_y(t)$. Expressing $K_y(t)$ as a series expansion, $K_y(t) = \sum_{j=1}^{\infty} K_j t^j / j!$, the coefficients K_j are the cumulants.
 - (a) Derive the cumulant generating function for the exponential dispersion family.
 - (b) Connect the first three cumulants to the first three moments.
3. Gamma distribution:
 - (a) Express the gamma distribution in the form of an exponential dispersion family, identifying the relevant components.
 - (b) What is the canonical link?
 - (c) Derive the mean and variance using the cumulant generating function.
4. Consider $Y_i \sim \text{Bin}(m_i, \pi_i), i = 1, \dots, n$. Assume the logit link is specified. Derive the likelihood equations, the asymptotic covariance matrix for $\hat{\beta}$, and the deviance.
5. Show that if we specify the link function to be the canonical link, then the Newton-Raphson and Fisher Scoring algorithms are identical (**Hint**: just need to show that expected and observed information matrices are the same for this case.)