FACTS and FORMULAS

- FACT: Suppose that $\mathbf{Y} \sim N(\mathbf{X}\boldsymbol{\beta}, \sigma^2\mathbf{I})$. Then, if $\mathbf{A}$ and $\mathbf{B}$ are matrices of constants,

$$E(\mathbf{AY}+\mathbf{B}) = \mathbf{AX}\boldsymbol{\beta}+\mathbf{B} \qquad \text{and} \qquad \text{var}(\mathbf{AY}+\mathbf{B}) = \sigma^2\mathbf{AA}'$$

- Least Squares related formulas

$$\begin{aligned} \mathbf{b} &= (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y} \\ \mathbf{H} &= \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}' \\ \hat{\mathbf{Y}} &= \mathbf{HY} \\ \mathbf{e} &= (\mathbf{I}-\mathbf{H})\mathbf{Y} \end{aligned}$$

- One-factor ANOVA

$$SSTO = SSTR + SSE$$

$$\sum_{i=1}^{r}\sum_{j=1}^{n_i}(Y_{ij}-\bar{Y}_{..})^2 = \sum_{i=1}^{r} n_i(\bar{Y}_{i.}-\bar{Y}_{..})^2 + \sum_{i=1}^{r}\sum_{j=1}^{n_i}(Y_{ij}-\bar{Y}_{i.})^2$$

- Two-factor ANOVA

$$SSTO = SSTR + SSE$$

$$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(Y_{ijk}-\bar{Y}_{...})^2 = n\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{Y}_{ij.}-\bar{Y}_{...})^2 + \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(Y_{ijk}-\bar{Y}_{ij.})^2$$

- Decomposition of Treatment SS

$$SSTR = SSA + SSB + SSAB$$

$$\begin{aligned} n\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{Y}_{ij.}-\bar{Y}_{...})^2 &= nb\sum_{i=1}^{a}(\bar{Y}_{i..}-\bar{Y}_{...})^2 + na\sum_{j=1}^{b}(\bar{Y}_{.j.}-\bar{Y}_{...})^2 \\ &\quad + n\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{Y}_{ij.}-\bar{Y}_{i..}-\bar{Y}_{.j.}+\bar{Y}_{...})^2 \end{aligned}$$

- Variance of a linear contrast

$$\text{var}\left(\sum_i c_i\bar{Y}_i\right) = \sigma^2\sum_i \frac{c_i^2}{n_i}$$