CHAPTER 2

21 (a) $P(X=k) = (1-p)^{k-1} p, \quad k = 1, 2, \ldots$

Hence, $P(X > m) = p \sum_{k=m+1}^{\infty} (1-p)^{k-1} = p(1-p)^m / \{1-(1-p)\}$

$= (1-p)^m, \quad m = 0, 1, 2, \ldots$

So, $P(X > n+k-1 \mid X > n-1)$

$= P(X > n+k-1, X > n-1) / P(X > n-1)$

$= P(X > n+k-1) / P(X > n-1) = (1-p)^{n+k-1} / (1-p)^{n-1}$

$= (1-p)^k = P(X > k)$.

(b) This is the so-called "lack of memory property" of the geometric distribution. Since the trials are independent Bernoulli, probability of k failures after a given number of failures is the same as that from the beginning of trials.

23. Prob. of r successes before kth failure

= Prob (r successes in the first r+k-1 trials and failure in the (r+k)th trial)

$= \binom{r+k-1}{r} p^r (1-p)^{k-1} \cdot (1-p) = \binom{r+k-1}{r} p^r (1-p)^k$

28. (i) $p_k = \binom{n}{k} p^k q^{n-k}$ $(q = 1-p)$. Hence, $p_0 = q^n$. Also,

$p_k = \frac{n!}{k!(n-k)!} p^k q^{n-k} = \frac{n!\, p^{k-1} q^{n-k+1}}{(k-1)!\,(n-k+1)!} \cdot \frac{n-k+1}{k} \cdot \frac{p}{q}$

$= p_{k-1} \frac{(n-k+1)p}{kq}$

(ii) $P(X \le 4) = P_0 + P_1 + P_2 + P_3 + P_4 \qquad n = 9000, \quad p = 0.0005$

$P_0 = (1-0.0005)^{9000} = 0.0111 \qquad P_1 = 0.04997$

$P_2 = 0.1125 \qquad P_3 = 0.1688 \qquad P_4 = 0.1899$

$P(X \le 4) = 0.5323$

30 (a) X: the number of suicides in a year

$X \sim \text{Poisson}(\lambda)$ $\lambda = 12 \times 0.33 = 0.396$

$P(X=k) = \frac{3.96^k}{k!} e^{-3.96}$ $k = 0, 1, 2, \cdots$

the most probable number of suicides is $k=3$, Since the mode of the Poisson distribution is the integer part of the rate.

(b) Z the number of suicides in one week (four weeks in one month)

$Z \sim \text{Poisson}(\lambda)$ $\lambda = \frac{0.33}{4} = 0.0825$

$P(Z=2) = \frac{0.0825^2}{2!} \cdot e^{-0.0825} = 0.0031$

44 Pdf of T is $f(x) = \begin{cases} \lambda \cdot e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$

$P(X=k) = P(k \leq T < k+1) = \int_k^{k+1} \lambda e^{-\lambda x} dx = -e^{-\lambda x}\Big|_k^{k+1}$

$= -e^{-\lambda(k+1)} - (-e^{-\lambda k}) = e^{-\lambda k}(1-e^{-\lambda})$ $k = 0, 1, 2, \cdots$

45 (a) T: the lifetime of an electronic component

$f(x) = \begin{cases} 0.1 e^{-0.1x} & x \geq 0 \\ 0 & x < 0 \end{cases}$

$P(T<10) = \int_0^{10} 0.1 e^{-0.1x} dx = -e^{-0.1x}\Big|_0^{10} = -e^{-1} + e^{0} = 1 - e^{-1}$

(b) $P(5 \leq T \leq 15) = \int_5^{15} 0.1 e^{-0.1x} dx = -e^{-0.1x}\Big|_5^{15} = -e^{-1.5} + e^{-0.5}$

(c) $P(T>t) = 0.01 = 1 - P(T \leq t) \Rightarrow P(T \leq t) = 0.99$

$\Rightarrow \int_0^t 0.1 e^{-0.1x} dx = 0.99 \Rightarrow 1 - e^{-0.1t} = 0.99 \Rightarrow t = 10 \cdot \ln 100 = 46.05$

Chapter 2

57 $X \sim N(\mu, \sigma^2)$ $\quad Y = aX + b \quad a<0$

$$f_Y(y) = f_X\left(\frac{y-b}{a}\right) \cdot \left|\frac{1}{a}\right| = \frac{1}{\sqrt{2\pi\sigma^2 a^2}} \cdot e^{-\frac{(y-a\mu-b)^2}{2\sigma^2 a^2}}$$

$Y \sim N(a\mu+b, a^2\sigma^2)$

(Use P_{60}. Proposition B)

67 $F(x) = 1 - e^{-\left(\frac{x}{\alpha}\right)^\beta}$ $\quad x \geq 0, \quad \alpha > 0, \quad \beta > 0$

(a) $f(x) = F'(x) = -e^{-\left(\frac{x}{\alpha}\right)^\beta} \cdot \left[-\beta \cdot \left(\frac{x}{\alpha}\right)^{\beta-1}\right] \cdot \frac{1}{\alpha} = \frac{\beta}{\alpha^\beta} \cdot x^{\beta-1} \cdot e^{-\left(\frac{x}{\alpha}\right)^\beta} \quad x \geq 0$

$$f(x) = \begin{cases} \frac{\beta}{\alpha^\beta} x^{\beta-1} e^{-\left(\frac{x}{\alpha}\right)^\beta} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

(b) $f_X(x) = f_W(\alpha \cdot x^{\frac{1}{\beta}}) \cdot \left|\frac{\alpha}{\beta} x^{\frac{1}{\beta}-1}\right| = \frac{\beta}{\alpha^\beta} \cdot \frac{\alpha}{\beta} \cdot x^{\frac{1}{\beta}-1} \cdot (\alpha \cdot x^{\frac{1}{\beta}})^{\beta-1} \cdot e^{-(x^{\frac{1}{\beta}})^\beta}$

$= e^{-x}$ $\quad \left(X = \left(\frac{W}{\alpha}\right)^\beta \Leftrightarrow W = \alpha \cdot X^{\frac{1}{\beta}} \right)$

this is pdf of exponential distribution with $\lambda = 1$

(c) $U \sim \text{Uniform}[0, 1]$ $\quad W = F^{-1}(U)$

$U = F(W) = 1 - e^{-\left(\frac{W}{\alpha}\right)^\beta} \Rightarrow W = F^{-1}(U) = \alpha \cdot [-\ln(1-U)]^{\frac{1}{\beta}}$

W follows a Weibull distribution