Chapter 8

8.6 (a) Geometric(p) $p = 0.358$

(c) $X = \sum_{i=1}^{12} \frac{(O_i - E_i)^2}{E_i} = 6.747$

$X \sim \chi^2_{10}$ P-value is 0.75

8.14 (a) $E|X| = \frac{1}{2\sigma}\int_{-\infty}^{\infty} |x| e^{-\frac{|x|}{\sigma}} dx = \frac{1}{\sigma}\int_0^{\infty} x \cdot e^{-\frac{x}{\sigma}} dx = \sigma$

$\hat{\sigma} = \frac{1}{n}\sum_{i=1}^{n} |X_i|$

(b) $lik(\sigma) = \prod_{i=1}^{n} \left[\frac{1}{2\sigma} e^{-\frac{|X_i|}{\sigma}}\right]$

$l(\sigma) = \sum_{i=1}^{n} \left[\ln 1 - \ln 2\sigma - \frac{|X_i|}{\sigma}\right] = -n \ln 2\sigma - \frac{1}{\sigma}\sum_{i=1}^{n} |X_i|$

$\frac{\partial l(\sigma)}{\partial \sigma} = -\frac{n}{\sigma} + \frac{1}{\sigma^2}\sum_{i=1}^{n} |X_i|$

Let $\frac{\partial l(\sigma)}{\partial \sigma} = 0 \Rightarrow \hat{\sigma}_{MLE} = \frac{\sum_{i=1}^{n} |X_i|}{n}$

8.16 (a) $\mu_2 = EX^2 = Var X + (EX)^2 = \frac{2}{9(3\alpha+1)} + \frac{1}{9} = \frac{\alpha+1}{9\alpha+3}$

$\Rightarrow \alpha = \frac{1 - 3\mu_2}{9\mu_2 - 1} \Rightarrow \hat{\alpha} = \frac{1 - \frac{3}{n}\sum_{i=1}^{n} X_i^2}{\frac{9}{n}\sum_{i=1}^{n} X_i^2 - 1}$

(b) $l(\alpha) = \sum_{i=1}^{n} \left[\ln\Gamma(3\alpha) - \ln\Gamma(\alpha) - \ln\Gamma(2\alpha) + (\alpha-1)\ln X_i + (2\alpha-1)\ln(1-X_i)\right]$

MLE of α will satisfy: $3\frac{\Gamma'(3\alpha)}{\Gamma(3\alpha)} - \frac{\Gamma'(\alpha)}{\Gamma(\alpha)} - 2\frac{\Gamma'(2\alpha)}{\Gamma(2\alpha)} + \sum_{i=1}^{n}\left[\ln X_i + 2\ln(1-X_i)\right]$

17 (a) $l(\mu,\sigma) = \sum_{i=1}^{n}\left[-\frac{1}{2}\ln 2\pi\sigma^2 - \frac{(X_i-\mu)^2}{2\sigma^2}\right]$

$= -\frac{n}{2}\ln 2\pi - \frac{n}{2}\ln\sigma^2 - \sum_{i=1}^{n}\frac{(X_i-\mu)^2}{2\sigma^2}$

when μ is known, set $\frac{\partial l}{\partial\sigma}=0 \Rightarrow \hat{\sigma}_{MLE}=\sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_i-\mu)^2}$

(b) if σ is known, set $\frac{\partial l}{\partial\mu}=0 \Rightarrow \hat{\mu}_{MLE} = \bar{X}$