9.12

likelihood ratio: $\Lambda = \dfrac{f(x_1,\cdots,x_n|\theta_0)}{\max\limits_{\theta>0} f(x_1,\cdots,x_n|\theta)}$

Find MLE of θ : $lik(\theta) = \theta^n e^{-\theta\sum_{i=1}^n x_i}$

$l(\theta) = \ln lik(\theta) = n\ln\theta - \theta\sum_{i=1}^n x_i$; $\frac{\partial l}{\partial\theta} = 0 \Rightarrow \hat{\theta} = \frac{1}{\bar{X}}$

SO $\Lambda = \dfrac{f(x_1,\cdots,x_n|\theta_0)}{f(x_1,\cdots,x_n|\hat{\theta})} = \dfrac{\theta_0^n e^{-\theta_0\sum_{i=1}^n x_i}}{(\frac{1}{\bar{X}})^n e^{-n}}$

Rejection region is $\{\Lambda \le k\} = \left\{\frac{\theta_0^n}{e^{-n}}\cdot(\bar{X})^n\cdot e^{-\theta_0\sum_{i=1}^n x_i} \le k\right\}$

$\Leftrightarrow \left\{\bar{X}\cdot e^{-\theta_0\bar{X}} \le \frac{k^{\frac{1}{n}}}{e\cdot\theta_0}\right\}$ let $\frac{k^{\frac{1}{n}}}{e\cdot\theta_0} = c$

So Rejection region is $\{\bar{X}e^{-\theta_0\bar{X}} \le c\}$

9.18 (a) T (b) F (c) T (d) F (e) F (f) F

9.26 Use Pearson's chi-square statistic

$X^2 = \sum_{i=1}^4 \frac{(O_i - E_i)^2}{E_i}$

$O_1 = 1997$; $O_2 = 906$; $O_3 = 904$, $O_4 = 32$

$N = O_1 + O_2 + O_3 + O_4 = 3839$

From Previous excercise, we know MLE of θ is 0.0357

SO $P_1 = 0.25(2+0.0357) = 0.5089$ $P_2 = 0.25(1-0.0357) = 0.2411$

$P_3 = 0.25(1-0.0357) = 0.2411$ $P_4 = 0.25\times 0.0357 = 0.0089$

$E_1 = NP_1 = 1953.67$ $E_2 = NP_2 = 925.58$ $E_3 = NP_3 = 925.58$

$E_4 = NP_4 = 34.26$ SO $X^2 = 2.054$

9.26

Degree of freedom is $4-1-1=2$

P-value : $P(\chi_2^2 > x^2) > 0.1$

So the model fits well

9.27

$$\chi^2 = \sum_{i=1}^{3} \frac{(O_i - E_i)^2}{E_i}$$

$O_1 = 10$ $\quad O_2 = 68$ $\quad O_3 = 112$

$N = O_1 + O_2 + O_3 = 190$

From previous excercise, MLE of θ is 0.768

So $P_1 = (1-0.768)^2 = 0.0538$ $\quad P_2 = 2 \times 0.768(1-0.768) = 0.3564$

$P_3 = (0.768)^2 = 0.5898$

$E_1 = NP_1 = 10.22$ $\quad E_2 = NP_2 = 67.71$ $\quad E_3 = NP_3 = 112.06$

So $\chi^2 = 0.0067$

Degree of freedom is $3-1-1=1$

P-value : $P(\chi_1^2 > x^2) = 0.9$

So the model fits well

9.33 (a) $\chi^2 = \sum_{i=1}^{2} \frac{(O_i - E_i)^2}{E_i}$

$O_1 = 9207 \quad O_2 = 8743 \quad N = O_1 + O_2 = 17950$

$E_1 = N \times \frac{1}{2} = 8975 \quad E_2 = N \times \frac{1}{2} = 8975$

$\chi^2 = 11.99$

Degree of freedom is : $2 - 1 = 1$

p-value : $P(\chi_1^2 > \chi^2) < 0.005$

So 9207 heads out of 17950 tosses is not consistent with the null hypothesis.

(b) $\chi^2 = \sum_{i=1}^{6} \frac{(O_i - E_i)^2}{E_i} \quad N = \sum_{i=1}^{6} O_i = 3590$

$O_1 = 100$; $O_2 = 524$; $O_3 = 1080$; $O_4 = 1126$; $O_5 = 655$; $O_6 = 105$

$P_1 = (\frac{1}{2})^5$; $P_2 = \binom{5}{1}(\frac{1}{2})^5$; $P_3 = \binom{5}{2}(\frac{1}{2})^5$; $P_4 = \binom{5}{3}(\frac{1}{2})^5$; $P_5 = \binom{5}{4}(\frac{1}{2})^5$; $P_6 = (\frac{1}{2})^5$

$E_1 = NP_1 = 112.19 \quad E_2 = NP_2 = 560.94 \quad E_3 = NP_3 = 1121.88$

$E_4 = NP_4 = 1121.88 \quad E_5 = NP_5 = 560.94 \quad E_6 = NP_6 = 112.19$

$\chi^2 = 21.57$

Degree of freedom is $6 - 1 = 5$

P-value : $P(\chi_5^2 > \chi^2) \approx 0.001$

So the data are not consistent with model

(C) $\chi^2 = \sum_{i=1}^{6} \frac{(O_i - E_i)^2}{E_i} \quad N = \sum_{i=1}^{6} O_i = 3590$

$O_1 = 100$, $N_2 = 524$, $O_3 = 1080$, $O_4 = 1126$, $O_5 = 655$, $O_6 = 105$

we use $9207/17950 = 0.513$ to estimate the probability of head.

So $P_1 = (1 - 0.513)^5 = 0.0274 \quad P_2 = \binom{5}{1} 0.513 (1 - 0.513)^4 = 0.1443$

$P_3 = \binom{5}{2} 0.513^2 (1 - 0.513)^3 = 0.304 \quad P_4 = \binom{5}{3} 0.513^3 (1 - 0.513)^2 = 0.3202$

$P_5 = \binom{5}{4} 0.513^4 (1 - 0.513) = 0.1686 \quad P_6 = \binom{5}{5} 0.513^5 = 0.0355$

3

$E_1 = NP_1 = 98.366$ $E_2 = NP_2 = 518.037$ $E_3 = NP_3 = 1091.36$

$E_4 = NP_4 = 1149.52$ $E_5 = NP_5 = 605.27$ $E_6 = NP_6 = 127.45$

$X^2 = 8.74$

Degree of freedom is $6-1-1=4$

P-value : $P(\chi_4^2 > X^2) \approx 0.07$

If $\alpha = 0.05$, we can not reject H_0, if $\alpha = 0.1$, we reject H_0