Chapter 8: Further Results

1. $\text{MSE}(\hat{\theta}) = E(\hat{\theta} - \theta)^2 = V(\hat{\theta}) + [E(\hat{\theta}) - \theta]^2$.

2. Given two estimators $\hat{\theta}$ and $\tilde{\theta}$ of θ, the efficiency of $\hat{\theta}$ relative to $\tilde{\theta}$ is given by $\text{eff}(\hat{\theta}, \tilde{\theta}) = \frac{\text{MSE}(\tilde{\theta})}{\text{MSE}(\hat{\theta})}$.

3. In particular, if both $\hat{\theta}$ and $\tilde{\theta}$ are unbiased estimators of θ, then $\text{eff}(\hat{\theta}, \tilde{\theta}) = \frac{V(\tilde{\theta})}{V(\hat{\theta})}$.

4. Under some regularity conditions, the Cramer-Rao lower bound for variances of unbiased estimators of θ is $1/[nI(\theta)]$, where $I(\theta) = E[\frac{\partial \log f_\theta(X)}{\partial \theta}]^2 = E[-\frac{\partial^2 \log f_\theta(X)}{\partial \theta^2}]$.

5. A statistic $T \equiv T(X_1, \cdots, X_n)$ is said to be sufficient for θ if the conditional distribution of $X_1, \cdots . X_n$ given $T = t$ does not depend on θ for any value of T.

6. Factorization Theorem: A necessary and sufficient condition for a statistic T to be sufficient for a parameter θ is that the joint pdf (or pf) factors in the form

$$f(x_1, \cdots, x_n | \theta) = g[T(x_1, \cdots, x_n), \theta] h(x_1, \cdots, x_n).$$