8.14 (d) $f(x_1, x_2, \cdots x_n|\sigma) = \prod_{i=1}^{n}\left[\frac{1}{2\sigma} e^{-\frac{|x_i|}{\sigma}}\right] = \left(\frac{1}{2\sigma}\right)^n \cdot e^{-\frac{1}{\sigma}\sum_{i=1}^{n}|x_i|}$

$g[T(x_1,\cdots,x_n), \sigma] = \left(\frac{1}{2\sigma}\right)^n e^{-\frac{1}{\sigma}\sum_{i=1}^{n}|x_i|}$ $\quad T(x_1,\cdots,x_n) = \sum_{i=1}^{n}|x_i|$

$h(x_1, \cdots, x_n) = 1$

sufficient statistic for σ is $\sum_{i=1}^{n}|x_i|$

8.16 (d) $f(x_1, x_2, \cdots x_n|\alpha) = \prod_{i=1}^{n}\left[\frac{\Gamma(3\alpha)}{\Gamma(\alpha)\Gamma(2\alpha)} x_i^{\alpha-1}(1-x_i)^{2\alpha-1}\right]$

$= \left(\frac{\Gamma(3\alpha)}{\Gamma(\alpha)\Gamma(2\alpha)}\right)^n \left[\prod_{i=1}^{n} x_i(1-x_i)^2\right]^{\alpha} \cdot \prod_{i=1}^{n}\frac{1}{x_i(1-x_i)}$

$h(x_1, \cdots, x_n) = \prod_{i=1}^{n}\frac{1}{x_i(1-x_i)}$

$T(x_1, \cdots, x_n) = \prod_{i=1}^{n}[x_i(1-x_i)^2]$

Sufficient Statistic for α is $\prod_{i=1}^{n}[x_i(1-x_i)^2]$

8.17 (c) Let $T(x_1, x_2, \cdots x_n)$ be any unbiased estimate of μ, from Cramer-Rao Inequality $Var(T) \geq \frac{1}{n I(\mu)}$

$I(\mu) = E\left[-\frac{\partial^2 \ln f(x|\mu)}{\partial \mu^2}\right] = E\left[-\frac{\partial^2}{\partial \mu^2}\left(\ln 1 - \ln \sigma\sqrt{2\pi} - \frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right)\right]$

$= E\frac{1}{\sigma^2} = \frac{1}{\sigma^2}$

$Var(T) \geq \frac{\sigma^2}{n} = Var(\bar{X})$

So, There is no other unbiased estimate of μ have Smaller variance than $\bar{X}$

8.54 (a) we know $T=\sum_{i=1}^{n} x_i \sim \text{Poisson}(n\lambda)$

$P(X_1=k_1, X_2=k_2, \dots X_n=k_n \mid T=t)$ when $\sum_{i=1}^{n} k_i \neq t$

this probability is 0 (doesn't depend on λ)

So, we assume $\sum_{i=1}^{n} k_i = t$

$$P(X_1=k_1, X_2=k_2, \cdots, X_n=k_n \mid T=t) = \frac{P(X_1=k_1, \cdots X_n=k_n, T=t)}{P(T=t)} = \frac{P(X_1=k_1, \cdots, X_n=k_n)}{P(T=t)}$$

$$= \frac{\frac{e^{-\lambda}\lambda^{k_1}}{k_1!} \times \cdots \times \frac{e^{-\lambda}\lambda^{k_n}}{k_n!}}{\frac{e^{-n\lambda}(n\lambda)^t}{t!}} = \frac{t!}{k_1! \cdots k_n! \cdot n^t}$$ (doesn't depend on λ)

so $T=\sum_{i=1}^{n} X_i$ is sufficient for λ

(b) $P(X_2=k_2 \mid X_1=k_1) = P(X_2=k_2) = \frac{e^{-\lambda}\lambda^{k_2}}{k_2!}$ (depend on λ)

so X_1 is not sufficient for λ

(c) $P(X_1=x_1, \cdots X_n=x_n \mid \lambda) = \prod_{i=1}^{n} \frac{e^{-\lambda}\lambda^{x_i}}{x_i!} = e^{-n\lambda} \cdot \lambda^{\sum_{i=1}^{n} x_i} \prod_{i=1}^{n} \frac{1}{x_i!}$

$h(x_1, \cdots, x_n) = \prod_{i=1}^{n} \frac{1}{x_i!}$; $g(T(x_1, \cdots, x_n), \lambda) = e^{-n\lambda} \cdot \lambda^{\sum_{i=1}^{n} x_i}$

$T(x_1, \cdots, x_n) = \sum_{i=1}^{n} x_i$ is sufficient for λ

55 $f(x_1, \cdots, x_n \mid p) = \prod_{i=1}^{n} p(1-p)^{x_i-1} = \left(\frac{p}{1-p}\right)^n \cdot (1-p)^{\sum_{i=1}^{n} x_i}$

$h(x_1, \cdots, x_n) = 1$ $T(x_1, \cdots, x_n) = \sum_{i=1}^{n} x_i$ is sufficient for P

57 $f(x_1, \cdots, x_n \mid \theta) = \prod_{i=1}^{n} \frac{\theta}{(1+x_i)^{\theta+1}} = \theta^n \left[\prod_{i=1}^{n} \frac{1}{1+x_i}\right]^{\theta+1}$

$$= \theta^n \left[\prod_{i=1}^{n}(1+x_i)\right]^{-\theta} \cdot \left[\prod_{i=1}^{n}(1+x_i)\right]^{-1}$$

$h(x_1, \cdots, x_n) = \left[\prod_{i=1}^{n}(1+x_i)\right]^{-1}$; $T(x_1, \cdots, x_n) = \prod_{i=1}^{n}(1+x_i)$ is a sufficient

Statistic for θ