Chapter 6: Summary

Chisquare(χ^2) Distribution: If $X_1, \cdots, X_n$ are iid N(0,1), then $V = \sum_{i=1}^n X_i^2$ is said to have a χ^2 distribution with n degrees of freedom (d.f.). It is symbolically written as χ_n^2.
The pdf of V is $f(v) = \frac{e^{-v/2} v^{n/2-1}}{2^{n/2}\Gamma(n/2)}$, $v > 0$. Note that this is a Gamma$(n/2, 1/2)$ pdf.
The mgf of χ_n^2 is $(1-2t)^{-n/2}$, $t < 1/2$.
$E(\chi_n^2) = n$, $V(\chi_n^2) = 2n$.

t-Distribution: Let Z and U be independently distributed with $Z \sim \mathrm{N}(0,1)$ and $U \sim \chi_n^2$. Then $T = Z/(U/n)^{1/2}$ is said to have Student's t distribution with n degrees of freedom. It is symbolically written as t_n.
The pdf is given by $f(t) = \frac{\Gamma(\frac{n+1}{2})}{\Gamma(\frac{n}{2})(n\pi)^{1/2}}(1 + \frac{t^2}{n})^{-(n+1)/2}$, $-\infty < t < \infty$.
The distribution is symmetric about zero. As $n \to \infty$, a t-density converges to a N(0,1) density.

F-Distribution: Let U and V be independently distributed with $U \sim \chi_m^2$ and $V \sim \chi_n^2$. Then $W = \frac{U/m}{V/n}$ is said to have an F-distribution with m, n degrees of freedom. It is symbolically written as $F_{m,n}$.
The pdf is given by $f(w) = \frac{\Gamma(\frac{m+n}{2})}{\Gamma(\frac{m}{2})\Gamma(\frac{n}{2})}(\frac{m}{n})^{m/2} w^{m/2-1}(1 + \frac{m}{n}w)^{-(m+n)/2}$.

Sample Mean and Sample Variance from a Normal Distribution: Let $X_1, \cdots, X_n$ $(n \geq 2)$ be iid $\mathrm{N}(\mu, \sigma^2)$. Define $\bar{X} = n^{-1}\sum_{i=1}^n X_i$ and $S^2 = (n-1)^{-1}\sum_{i=1}^n (X_i - \bar{X})^2$. Then
(a) $\bar{X} \sim \mathrm{N}(\mu, \sigma^2/n)$.
(b) $\bar{X}$ and S^2 are independently distributed.
(c) $(n-1)S^2/\sigma^2 \sim \chi_{n-1}^2$.
(d) $n^{1/2}(\bar{X} - \mu)/S \sim t_{n-1}$.