11.1 c $n=4$, $m=5$ $S_X^2 = 0.216$, $S_Y^2 = 1.179$

$\bar{X}=0.5546$; $\bar{Y}=1.624$, $S_p^2 = \frac{3S_X^2+4S_Y^2}{7} = 0.7664$; $S_p = 0.8755$

d 90% CI is $(\bar{X}-\bar{Y}) \pm 1.895 \times 0.8755\sqrt{\frac{1}{4}+\frac{1}{5}}$ $\boxed{S_p\sqrt{\frac{1}{4}+\frac{1}{5}} \approx 0.5873}$

$\Rightarrow (-2.1823, 0.0435)$

e Use two-sided test

f $t = \frac{\bar{X}-\bar{Y}}{S_p\sqrt{\frac{1}{4}+\frac{1}{5}}} = -1.821$; P-value $= P(t_7 > |1.821| \text{ or } t_7 < -|1.821|) > 0.1$

g $\because$ P-value $> \alpha = 0.1$ so we can not reject $H_0: \bar{X} = \bar{Y}$

11.2 $Var(\bar{X}-\bar{Y}) = \frac{\sigma^2}{n} + \frac{\sigma^2}{n} = \frac{2\sigma^2}{n}$

when $\sigma_1 = \frac{\sigma}{2}$ $Var(\bar{X}-\bar{Y}) = \frac{2\cdot\frac{\sigma^2}{4}}{n} = \frac{\sigma^2}{2n}$

when $n_1 = 2n$ $Var(\bar{X}-\bar{Y}) = \frac{2\sigma^2}{n_1} = \frac{2\sigma^2}{2n} = \frac{\sigma^2}{n}$

$\frac{\sigma^2}{2n} < \frac{\sigma^2}{n}$

So to halve the standard deviations is better than double the sample size.

11.3 when $n=m$ then $S_p^2 = \frac{(n-1)S_X^2+(m-1)S_Y^2}{m+n-2} = \frac{(n-1)(S_X^2+S_Y^2)}{2(n-1)}$

$= \frac{S_X^2+S_Y^2}{2}$

so $S_p^2\left(\frac{1}{n}+\frac{1}{m}\right) = \frac{S_X^2+S_Y^2}{n}$

$\frac{S_X^2}{n} + \frac{S_Y^2}{m} = \frac{S_X^2+S_Y^2}{n}$

so these two estimates are identical if $m=n$

11.30 $X \sim N(\mu_X, \sigma_X^2)$ and Y is independent $N(\mu_Y, \sigma_Y^2)$

So $X-Y \sim N(\mu_X-\mu_Y, \sigma_X^2+\sigma_Y^2)$

$\pi = P(X<Y) = P(X-Y<0) = P\left(\frac{X-Y-(\mu_X-\mu_Y)}{\sqrt{\sigma_X^2+\sigma_Y^2}} < \frac{0-(\mu_X-\mu_Y)}{\sqrt{\sigma_X^2+\sigma_Y^2}}\right)$

$= \Phi\left(\frac{\mu_Y-\mu_X}{\sqrt{\sigma_X^2+\sigma_Y^2}}\right)$ $\Phi(\cdot)$ is cdf of standard Normal

1.31 we know $\frac{(n-1)S_X^2}{\sigma_X^2} \sim \chi^2_{n-1}$; $\frac{(m-1)S_Y^2}{\sigma_Y^2} \sim \chi^2_{m-1}$

$\frac{(n-1)S_X^2}{\sigma_X^2}$ and $\frac{(m-1)S_Y^2}{\sigma_Y^2}$ are independent

So $\frac{S_X^2/\sigma_X^2}{S_Y^2/\sigma_Y^2} = \frac{S_X^2}{S_Y^2} \cdot \frac{\sigma_Y^2}{\sigma_X^2} \sim F_{n-1,\, m-1}$

when $\sigma_X = \sigma_Y \Rightarrow \frac{S_X^2}{S_Y^2} \sim F_{n-1,\, m-1}$

(a) For one-sided test H_a: $\frac{\sigma_X}{\sigma_Y} > 1$

reject if $\frac{S_X^2}{S_Y^2} > F_{n-1,\, m-1}(\alpha)$

For two-sided test H_a: $\frac{\sigma_X}{\sigma_Y} \neq 1$

reject if $\frac{S_X^2}{S_Y^2} > F_{n-1,\, m-1}(\frac{\alpha}{2})$ or $\frac{S_X^2}{S_Y^2} < F_{n-1,\, m-1}(1-\frac{\alpha}{2})$

(b) because $\frac{S_X^2}{S_Y^2} \cdot \frac{\sigma_Y^2}{\sigma_X^2} \sim F_{n-1,\, m-1}$

So $P\left(F_{n-1,\, m-1}^{(1-\frac{\alpha}{2})} < \frac{S_X^2}{S_Y^2} \cdot \frac{\sigma_Y^2}{\sigma_X^2} < F_{n-1,\, m-1}^{(\frac{\alpha}{2})}\right) = 1-\alpha$

$\Leftrightarrow P\left(\frac{F_{n-1,m-1}(1-\frac{\alpha}{2})}{S_X^2/S_Y^2} < \frac{\sigma_Y^2}{\sigma_X^2} < \frac{F_{n-1,m-1}(\frac{\alpha}{2})}{S_X^2/S_Y^2}\right) = 1-\alpha$

$\Leftrightarrow P\left(\frac{S_X^2/S_Y^2}{F_{n-1,m-1}(\frac{\alpha}{2})} < \frac{\sigma_X^2}{\sigma_Y^2} < \frac{S_X^2/S_Y^2}{F_{n-1,m-1}(1-\frac{\alpha}{2})}\right) = 1-\alpha$

So $100(1-\alpha)\%$ Confidence interval for $\frac{\sigma_X^2}{\sigma_Y^2}$ is $\left[\frac{S_X^2/S_Y^2}{F_{n-1,m-1}(\frac{\alpha}{2})}, \frac{S_X^2/S_Y^2}{F_{n-1,m-1}(1-\frac{\alpha}{2})}\right]$

(c) $S_A^2/S_B^2 = 0.6$, The p-value for a two-sided test is 0.42

$(P(F_{12,7} < 0.6) = 0.42)$

So can not reject H_0: $\sigma_A^2 = \sigma_B^2$

A 95% CI for $\frac{\sigma_A^2}{\sigma_B^2}$ is $(0.13, 2.16)$