An MCMC Approach to Empirical Bayes Inference and Bayesian Sensitivity Analysis via Empirical Processes

Hani Doss
University of Florida

Consider a Bayesian situation in which we observe $Y \sim p_\theta$, where $\theta \in \Theta$, and we have a family $\{\nu_h,\, h \in \mathcal{H}\}$ of potential prior distributions on Θ. Let g be a real-valued function of θ, and let $I_g(h)$ be the posterior expectation of $g(\theta)$ when the prior is ν_h. We are interested in two problems: (i) selecting a particular value of h, and (ii) estimating the family of posterior expectations $\{I_g(h),\, h \in \mathcal{H}\}$. Let $m_y(h)$ be the marginal likelihood of the hyperparameter h: $m_y(h) = \int p_\theta(y)\, \nu_h(d\theta)$. The empirical Bayes estimate of h is, by definition, the value of h that maximizes $m_y(h)$. It turns out that it is typically possible to use Markov chain Monte Carlo to form point estimates for $m_y(h)$ and $I_g(h)$ for each individual h in a continuum, and also confidence intervals for $m_y(h)$ and $I_g(h)$ that are valid pointwise. However, we are interested in forming estimates, with confidence statements, of the entire families of integrals $\{m_y(h),\, h \in \mathcal{H}\}$ and $\{I_g(h),\, h \in \mathcal{H}\}$: we need estimates of the first family in order to carry out empirical Bayes inference, and we need estimates of the second family in order to do Bayesian sensitivity analysis. We establish strong consistency and functional central limit theorems for estimates of these families by using tools from empirical process theory. As an application, we consider Latent Dirichlet Allocation, which is heavily used in topic modelling. The two-dimensional hyperparameter that governs this model has a large impact on inference. We show how our methodology can be used to select it. We also show how to form globally-valid "two-dimensional confidence bands" for certain posterior probabilities of interest, and give an illustration on a corpus consisting of a set of articles from Wikipedia.

Joint work with Yeonhee Park (MD Anderson Cancer Center).