

# STA 4183 – The Theory of Interest

## Formulas for Bonds

Bonds	$K = C \left( \frac{1}{1+i} \right)^n$	$g = \frac{Fr}{C}$	$G = \frac{Fr}{i}$
	$P = Fra_{\overline{n} i} + C \left( \frac{1}{1+i} \right)^n$	$P = C + C(g - i)a_{\overline{n} i}$	
	$P = G + (C - G) \left( \frac{1}{1+i} \right)^n$	$P = K + \frac{g}{i}(C - K)$	

Bond Amortization Schedule	
$I_t = iB_{t-1}$	$I_t = Ci + Ci(g - i)a_{\overline{n-t+1} i}$
$P_t = Fr - I_t$	$P_t = C(g - i) \left( \frac{1}{1+i} \right)^{n-t+1}$
$B_t = B_{t-1} - P_t$	$B_t = C + C(g - i)a_{\overline{n-t} i}$

	$B_{t+k}^f$	$Fr_k$	$B_{t+k}^m$
Theoretical	$B_t(1+i)^k$	$Fr \left( \frac{(1+i)^k - 1}{i} \right)$	$B_t(1+i)^k - Fr \left( \frac{(1+i)^k - 1}{i} \right)$
Practical	$B_t(1+ki)$	$kFr$	$B_t(1+ki) - kFr$
Semi-theoretical	$B_t(1+i)^k$	$kFr$	$B_t(1+i)^k - kFr$

Yield rate $i$
$i \approx \frac{g - \frac{k}{n}}{1 + \left( \frac{n+1}{2n} \right) k}$ where $k = \frac{P - C}{C}$

Yield rate $i'$ Reinvestment rate $j$
$(1+i')^n = \frac{C}{P} + \frac{Fr}{P} s_{\overline{n} j}$