STA 4183 – The Theory of Interest

Formulas for Varying Annuities

Increasing Annuities	Present value	Accumulated value
Immediate	$(Ia)_{\overline{n}\vert} = \dfrac{\ddot{a}_{\overline{n}\vert} - n\left(\frac{1}{1+i}\right)^n}{i}$	$(Is)_{\overline{n}\vert} = \dfrac{\ddot{s}_{\overline{n}\vert} - n}{i}$
Due	$(I\ddot{a})_{\overline{n}\vert} = \dfrac{\ddot{a}_{\overline{n}\vert} - n\left(\frac{1}{1+i}\right)^n}{d}$	$(I\ddot{s})_{\overline{n}\vert} = \dfrac{\ddot{s}_{\overline{n}\vert} - n}{d}$

Increasing Perpetuities	Present value
Immediate	$(Ia)_{\overline{\infty}\vert} = \dfrac{1}{id}$
Due	$(I\ddot{a})_{\overline{\infty}\vert} = \dfrac{1}{d^2}$

Decreasing Annuities	Present value	Accumulated value
Immediate	$(Da)_{\overline{n}\vert} = \dfrac{n - a_{\overline{n}\vert}}{i}$	$(Ds)_{\overline{n}\vert} = \dfrac{n\,(1+i)^n - s_{\overline{n}\vert}}{i}$
Due	$(D\ddot{a})_{\overline{n}\vert} = \dfrac{n - a_{\overline{n}\vert}}{d}$	$(D\ddot{s})_{\overline{n}\vert} = \dfrac{n\,(1+i)^n - s_{\overline{n}\vert}}{d}$

Geometric Annuities	Present value	Accumulated value
Immediate	$(Ga)_{\overline{n}\vert} = \dfrac{1 - \left(\frac{1+x}{1+i}\right)^n}{i - x}$	$(Gs)_{\overline{n}\vert} = \dfrac{(1+i)^n - (1+x)^n}{i - x}$
Due	$(G\ddot{a})_{\overline{n}\vert} = \dfrac{1 - \left(\frac{1+x}{1+i}\right)^n}{\left(\frac{i-x}{1+i}\right)}$	$(G\ddot{s})_{\overline{n}\vert} = \dfrac{(1+i)^n - (1+x)^n}{\left(\frac{i-x}{1+i}\right)}$

Geometric Perpetuities	Present value
Immediate	$(Ga)_{\overline{\infty}\vert} = \dfrac{1}{i - x}$ for $i > x$
Due	$(G\ddot{a})_{\overline{\infty}\vert} = \dfrac{1+i}{i - x}$ for $i > x$