

**Errata for *Statistical Inference, Second Edition* (Seventh Printing)**

Last Update: May 15, 2007

These errata were compiled for the seventh printing. Previous errata have already been incorporated in past printings. We have also been informed of errata in the solutions manual, but have not yet addressed that. Caveat Emptor!

Title Page “North Carolina State University” should be “Arizona State University”

page 43) Exercise 1.42(b): Replace part (b) with the following‘:

Let  $E$  be an arbitrary sample point in  $E_k$ . Without loss of generality, assume that  $E$  is contained in  $A_1, A_2, \dots, A_k$ . Show that  $P(E)$  appears  $k$  times in the sum  $P_1$ ,  $\binom{k}{2}$  times in the sum  $P_2$ ,  $\binom{k}{3}$  times in the sum  $P_3$ , etc.

page 43) Exercise 1.43(b): Delete the phrase “the  $P_i$  satisfy  $P_i > P_j$  if  $i < j$  and that”

page 45) line 10 ↑: Delete the phrase “the  $P_i$  are ordered in that  $P_i > P_j$  if  $i < j$ , and ”

page 129) line 6 ↑: “1.23 ” should be “1.28”

page 181) line 6 ↑: “... ,  $n$ ” should be “... ,  $m$ ”

page 182) line 10 ↑: Replace the display with

$$\text{Cov}(X_i, X_j) = E[(X_i - mp_i)(X_j - mp_j)] = -mp_i p_j.$$

page 195) Exercise 4.29(b): Replace

”Show that the distribution of  $(2XY)/\sqrt{X^2 + Y^2}$  is the same as the distribution of  $X$ . Specialize this result to one about  $n(0, \sigma^2)$  random variables” with

”If  $\theta$  and  $R$  are independent, show that  $(2XY)/\sqrt{X^2 + Y^2}$  and  $X$  have the same distribution. Specialize this result to one about  $n(0, \sigma^2)$  random variables”

page 219) line 6 ↓: “  $1/n$ ” should be “ $n$ ”

page 245) line 7 ↓: “ $\bar{X}$  ” should be “ $X$ ”

page 245) line 13 ↓: “ $\sum_{k=1}^n$  ” should be “ $\frac{1}{n} \sum_{k=1}^n$  ”

page 257) line 10 ↓: “squared correlation coefficient ” should be “squared sample correlation coefficient”

page 265) Exercise 5.61(b): “ $V \sim \text{gamma}([a], b)$  ” should be “  $V \sim \text{gamma}([a], b+1)$ ”

page 289) line 11 ↑: Replace the statement of Theorem 6.2.28 with

*If a minimal sufficient statistic exists, then any complete sufficient statistic is also a minimal sufficient statistic.*

page 297) line 16 ↑: “equivariant” should be “invariant”

page 305) Exercise 6.37(b)(i): “[ $2n/(2n + 1)$ ] $I(\theta)$ ” should be “[ $n/(2n + 1)$ ] $I(\theta)$ ”

page 309) line 3 ↑: “6.26” should be “6.29”

page 355) Exercise 7.2: Replace

- (a) Find the MLE of  $\beta$ , assuming  $\alpha$  is known.
- (b) If  $\alpha$  and  $\beta$  are both unknown, there is no explicit formula for the MLEs of  $\alpha$  and  $\beta$ , but the maximum can be found numerically. The result in part (a) can be used to reduce the problem to the maximization of a univariate function. Find the MLEs for  $\alpha$  and  $\beta$  for the data in Exercise 7.10(c).

with

- (a) Find the MLE of  $\beta$ , assuming  $\alpha$  is known.
- (b) Show that, if both  $\alpha$  and  $\beta$  are unknown,  $\bar{X}$  is the MLE of  $\alpha\beta$ .
- (c) If  $\alpha$  and  $\beta$  are both unknown, there is no explicit formula for the MLEs of  $\alpha$  and  $\beta$ , but the maxima can be found numerically. Use part (a) to reduce the problem to the maximization of a univariate function. Find the MLEs for  $\alpha$  and  $\beta$  for the data in Exercise 7.10(c).

page 368 – 369) Replace

The answer is no. That is, posterior means are *never* unbiased estimators. If they were, then taking the expectation over the joint distribution of  $X$  and  $\theta$ , we could write

$$\begin{aligned} E[(X - \theta)^2] &= E[X^2 - 2X\theta + \theta^2] && \text{(expand the square)} \\ &= E(E((X^2 - 2X\theta + \theta^2)|\theta)) && \text{(iterate the expectation)} \\ &= E(E(X^2|\theta) - 2\theta^2 + \theta^2) && (E(X|\theta) = E_\theta X = \theta) \\ &= E(E(X^2|\theta) - \theta^2) \\ &= E(X^2) - E(\theta^2), && \text{(properties of expectations)} \end{aligned}$$

doing the conditioning one way, and conditioning on  $X$  we could similarly calculate

$$\begin{aligned} E[(X - \theta)^2] &= E(E[(X^2 - 2X\theta + \theta^2)|X]) \\ &= E(X^2 - 2X^2 + E(\theta^2|X)) && \left( \begin{array}{l} E(\theta|X) = X \\ \text{by assumption} \end{array} \right) \\ &= E(\theta^2) - E(X^2). \end{aligned}$$

Comparing the two calculations, we see that the only way that there is no contradiction is if  $E(X^2) = E(\theta^2)$ , which then implies that  $E(X - \theta)^2 = 0$ ,

so  $X = \theta$ . This occurs only if  $P(X = \theta) = 1$ , an interesting situation, so we have argued to a contradiction.

with

The answer is no. That is, posterior means are not unbiased estimators except in a degenerate case, as we will see.

Suppose that

- (1)  $T$  is the posterior mean, so  $T = E(\theta|X)$ ,
- (2)  $T$  is unbiased, so  $E_\theta T = \theta$ .

Taking the expectation over the joint distribution of  $X$  and  $\theta$  and using the facts that  $T$  is a function of  $X$  and is the posterior mean we have

$$E(\theta T) = EE(\theta T|X) = E(TE(\theta|X)) = E(TT) = ET^2.$$

Conditioning on  $\theta$  and using the fact that  $T$  is unbiased yields

$$E(\theta T) = EE(\theta T|\theta) = E(\theta E(T|\theta)) = E(\theta\theta) = E\theta^2.$$

Using these two equalities we obtain

$$\begin{aligned} E(T - \theta)^2 &= E(T^2 - 2\theta T + \theta^2) \\ &= (ET^2 - E(\theta T)) + (E\theta^2 - E(\theta T)) \\ &= 0. \end{aligned}$$

This implies  $P(T = \theta) = 1$ , an uninteresting situation. The only way that  $T$  can be both unbiased and the Bayes estimator is if we have this degenerate situation.

page 391) last line :Replace

$$T > t_0 \Leftrightarrow \frac{g(t|\theta')}{g(t|\theta_0)} > k'.$$

with

$$\left\{ t : \frac{g(t|\theta')}{g(t|\theta_0)} > k' \right\} \subset \{t : t > t_0\} \subset \left\{ t : \frac{g(t|\theta')}{g(t|\theta_0)} \geq k' \right\}.$$

page 422) first line : “is a  $1 - \alpha$  confidence set. ” should be “has confidence coefficient  $\geq 1 - \alpha$ .”

page 434) line 14 ↓: Replace

$$\{\theta : F_T(T|\theta) \leq \alpha_1 \text{ and } \bar{F}_T(T|\theta) \leq \alpha_2\}$$

with

$$\{\theta : F_T(T|\theta) \geq \alpha_1 \text{ and } \bar{F}_T(T|\theta) \geq \alpha_2\}$$

page 434) line 18 ↓: Replace

$$\begin{aligned}\theta > \theta_U(t) &\Rightarrow F_T(t|\theta) < \frac{\alpha}{2}, \\ \theta < \theta_L(t) &\Rightarrow \bar{F}_T(t|\theta) < \frac{\alpha}{2},\end{aligned}$$

and hence  $\{\theta : F_T(T|\theta) \leq \alpha_1 \text{ and } \bar{F}_T(T|\theta) \leq \alpha_2\} = \{\theta : \theta_L(T) \leq \theta \leq \theta_U(T)\}$ .

with

$$\begin{aligned}\theta > \theta_U(t) &\Rightarrow F_T(t|\theta) < \alpha_1, \\ \theta < \theta_L(t) &\Rightarrow \bar{F}_T(t|\theta) < \alpha_2,\end{aligned}$$

and hence  $\{\theta : F_T(T|\theta) \geq \alpha_1 \text{ and } \bar{F}_T(T|\theta) \geq \alpha_2\} = \{\theta : \theta_L(T) \leq \theta \leq \theta_U(T)\}$ .

page 454) line 6 ↑: “beta( $x, n - x + 1$ ) ” should be “beta( $x + 1, n - x$ )”

page 469) line 7 ↑: Replace

We close this section with the outline of a more general result concerning the consistency of maximum likelihood estimators. This result shows that MLEs are consistent estimators of their parameters, and is the first case we have seen in which a method of finding an estimator guarantees an optimality property.

with

We close this section with the outline of a more general result concerning the asymptotic properties of MLEs. Formally, these results only apply to a *consistent root* of the likelihood equation, which may not be the MLE . We take a bit of liberty here, and discuss the results in terms of the MLE (see Lehmann and Casella 1998, Section 6.3 for the full story).

page 472) line 13 ↓: “the zero of the likelihood function. ” should be “a solution of  $l'(\theta|x) = 0$ .”

page 472) Display (10.1.6): Delete the minus sign in the first line of the display, and add a minus sign at the beginning of the second line of the display.

page 474) line 2 ↑: “Exercise ” should be “Example”

page 477) line 8 ↓:

“ Difficult as it may seem to believe, estimation of the mean of a gamma distribution is not an easy task.”

should be

“Surprisingly, if  $\beta$  is known, estimation of the mean of a gamma distribution is not an easy task.”

page 477) line 10 ↑:

“To ease the computations, assume that  $\beta$  is known so we solve  $\frac{d}{d\mu}l(\mu, \beta|\mathbf{x}) = 0$  to get the MLE  $\hat{\mu}$ . There is no explicit solution—so we proceed numerically.”

should be

“Since we are assuming that  $\beta$  is known, we solve  $\frac{d}{d\mu}l(\mu, \beta|\mathbf{x}) = 0$  to get the MLE  $\hat{\mu}$ . There is no explicit solution—so we proceed numerically.”

page 482) line 11 ↑: “usually ” should be “unusually”

page 508) line 7 ↓: “Exercise ” should be “Example”

page 508) line 17 ↓: “Exercise ” should be “Example”

page 516) line 15 ↓:

“Be forewarned, the following is not for the fainthearted, and can be skipped without sacrificing much in the way of understanding. ”

should be

“That is, it is important to realize that these theorems apply to estimators that are obtained as solutions to the equation  $l'(\theta|x) = 0$ .”

page 516) line 15 ↑: Delete “of MLEs”

page 516) line 8 ↑: Delete “of MLEs”

page 641) Reference 267: “*Principles of Real Analysis* ” should be “*Principles of Mathematical Analysis* ”