Lecture 19 Monday February 27, 2023

Majors : Paper by Kullgren et al
Option 1 for ALC Project
(last hw) is to
review this paper
for the statistical content

PLAN

- Lecture notes pp. 97 - 103
- Example to follow on Project 1 Q#5

Example, to imitate for Project 1 Q# 5

Below is the R code for obtaining the plot of results from Tukey's studentized range method of all pairwise comparisons, followed by the graph, which shows the six pairwise simultaneous 95% confidence intervals.

[In Project 1 you need to provide the numerical endpoints of the CIs, not the graph.]

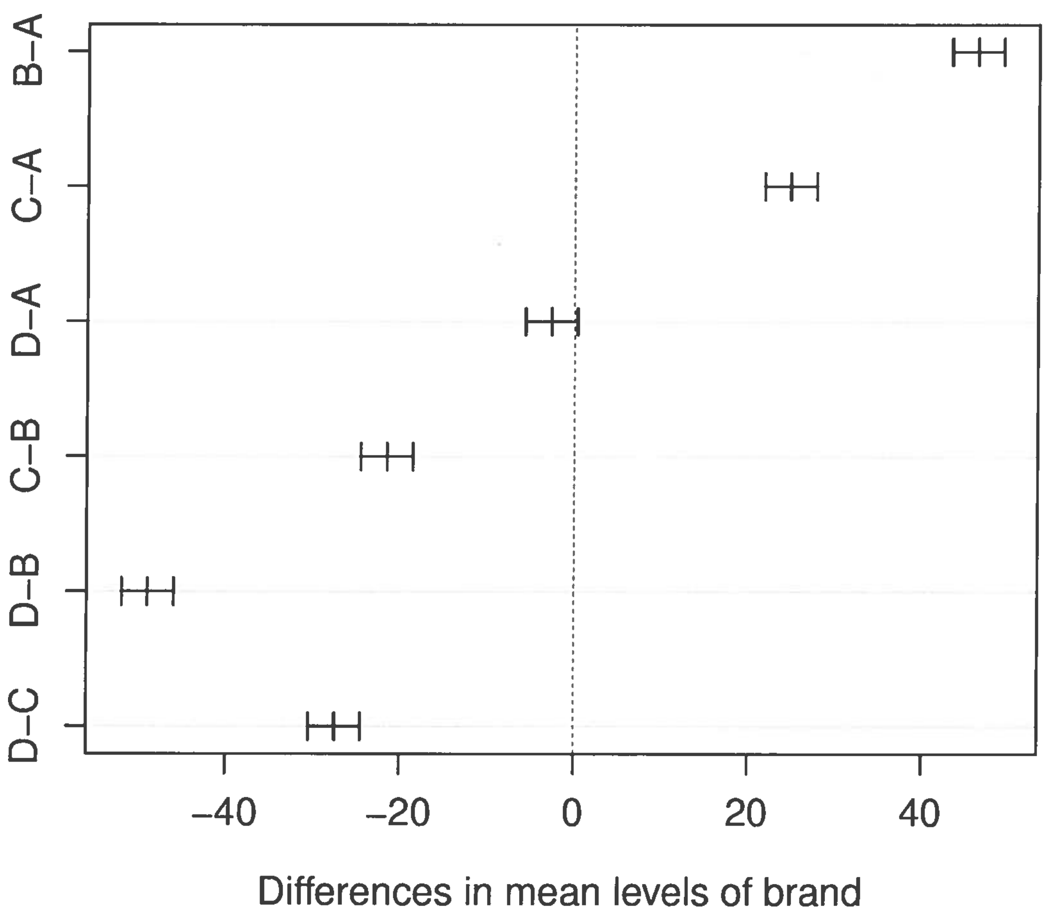
Tukey's "honest significant difference"

```
> plot(TukeyHSD(rust.aov))
```

__Note__ To get the numbers for these CIs, use

```
> TukeyHSD(rust.aov)
```

~~95% family-wise confidence level~~ (Use title on next page)



$\mu_B - \mu_A$

$\mu_C - \mu_A$

$\mu_D - \mu_A$

0 vertical line indicates "not a significant difference"

Table 2. Simultaneous 95% confidence intervals for all pairwise comparisons by Tukey's method

Parameter	Comparison	Estimate	SE	Interval
D_1	$\mu_B - \mu_A$	46	1.1	(43, 49)
D_2	$\mu_C - \mu_A$		1.1	
D_3	$\mu_A - \mu_D$		1.1	
D_4	$\mu_B - \mu_C$		1.1	
D_5	$\mu_B - \mu_D$		1.1	
D_6	$\mu_C - \mu_D$		1.1	

Here are the details for the ingredients for estimating D_1. Since the design is balanced, we have $n_i = n = 10$ for $i = 1, 2, 3, 4$, and all six SEs (estimated standard deviations of the comparison estimates) will be equal.

For D_1, we find $\hat{D}_1 = \bar{Y}_{B.} - \bar{Y}_{A.} = 89.44 - 43.14 = 46.3$.

And, $s(\hat{D}_1) = \sqrt{\text{MSE}\frac{2}{10}} = \sqrt{6.1\frac{2}{10}} = 1.1045$.

For 95% confidence with Tukey's method, the critical constant is 2.693227. This is given by the formula $1/\sqrt{2} \times q(.95, r = 4, df = 36)$, and was found in R by (fill in the blank).

The allowance for the confidence interval is $1.1045 \times 2.693227 = 2.9747$, or about 3.

CIs : Estimate $\pm$ 3

Interpretation of family-wise confidence level in the example:

We are 95% confident that all of the six confidence intervals are correct (contain the parameter being esimated).

Wordier explanation:

In infinitely many repeat experiments like this one, with the same number of groups and sample sizes, and where we form the six intervals by Tukey's method, the proportion of experiments with all intervals correct would be 95%.

Simultaneous Testing

Form the family of confidence intervals (CIs) for all pairwise comparisons by Tukey's method, at level $100(1-\alpha)\%$. For each of the $\binom{r}{2}$ tests of the hypothesis of the form $H_0 : \mu_I - \mu_j = 0$, reject H_0 if and only if the CI for that parameter does not include zero. The *familywise Type I error rate* of this procedure is α.

Ex. Rust data Conclusions from Tukey's that would be appropriate for Project 1 Q#5

We are 95% confident that each of the six confidence intervals is correct, that is, that it contains the parameter being estimated.

Since Inhibitor B has the highest observed mean, and since each of the three CIs comparing B to another inhibitor does not contain 0, we conclude that B is superior to all three of the other inhibitors. Also, the two worst-performing inhibitors—A and D— do not differ significantly from each other.

Note about Tukey's method in Project 1: The group sample sizes are very similar to each other but not all identical. In this case, Tukey's method may still be used, but the standard errors of the parameter estimates will not all be equal. We have instead, for $D = \mu_1 - \mu_2$, the usual formula for $s(\hat{D})$ with unequal sample sizes: $s(\hat{D}) = \sqrt{\text{MSE}(1/n_1 + 1/n_2)}$ The estimate of D will be the same as before: Tukey's critical constant: $1/\sqrt{2} \times q(.95, r, n_T - r)$

$\bar{Y}_{1.} - \bar{Y}_{2.}$

Scheffé multiple comparison method

The Scheffé method allows simultaneous confidence intervals (or tests) for all possible contrasts of the cell means,

$$L = \sum_{i=1}^{r} c_i \mu_i, \text{ where } \sum_{i=1}^{r} c_i = 0$$

Procedure:

The estimate of L is:

$$\hat{L} = \sum_{i=1}^{r} c_i \overline{Y}_{i.}$$

Estimate the variance of $\hat{L}$ by

$$\widehat{Var}(\hat{L}) = s^2(\hat{L}) = \text{MSE} \sum \frac{c_i^2}{n_i}$$

Let $S^2 = (r-1)\underbrace{F(1-\alpha; r-1, n_T - r)}_{\text{critical constant for F test of } H_0: \mu_1 = \dots = \mu_r}$ (Here the capital "S" refers to critical constant for Scheffé method)

Scheffé critical constant:

Let $S^2 = (r-1)F(1-\alpha; r-1, n_T - r)$ (Here the capital "S" refers to critical constant for Scheffé method)

The Scheffé confidence intervals are:

$$\hat{L} \pm \underbrace{S}_{\text{critical constant}}\ \underbrace{s(\hat{L})}_{\text{est. SD of distn of } \hat{L}}$$

Ex Rust, all pairwise comparisons

Purpose of this example is to compare Scheffé method to Tukey's studentized range method, in the case where both apply.

Tukey: Let $\hat{D} = \bar{Y}_{i.} - \bar{Y}_{j.}$

$$\hat{D} \pm \frac{1}{\sqrt{2}} q(1-\alpha, r, n_T - r)\text{SE}(\overline{Y}_{i.} - \overline{Y}_{j.})$$

Scheffé:

$$\hat{D} \pm \sqrt{(r-1)F(1-\alpha; r-1, n_T - r)}\,\text{SE}(\overline{Y}_{i.} - \overline{Y}_{j.})$$

Rust example Find the critical constants for: individual 95% t CI, Tukey's method, and Scheffé method

```
> qt(.975,36)
[1] 2.028094
> qtukey(.95,nmeans=4,df=36)/sqrt(2)
[1] 2.693227
> S <- sqrt(3*qf(.95,3,36)); S
[1] 2.93237
```

Scheffé method can handle any contrast, not just differences of two means.

Ex Suppose you wanted to compare Brands A and D to Brands B and C, so you decide to estimate the contrast

worst inhibitors   best two inhibitors

$$L = \frac{\mu_A + \mu_D}{2} - \frac{\mu_B + \mu_C}{2}$$

The estimate is:

$$\hat{L} = \frac{\overline{Y}_{\text{A.}} + \overline{Y}_{\text{D.}}}{2} - \frac{\overline{Y}_{\text{B.}} + \overline{Y}_{\text{C.}}}{2}$$

and the estimate of the variance is

$$\hat{\text{Var}}(\hat{L}) = \frac{1}{4}\left(4 \times \frac{\text{MSE}}{n}\right) = \frac{\text{MSE}}{n}$$

Scheffé 95% CI for $L = \frac{\mu_A + \mu_D}{2} - \frac{\mu_B + \mu_C}{2}$ in the Rust data

$S = \sqrt{3 \times F_{.95,3,36}} = 2.93237$

Estimate: $\hat{L} = \frac{43.14 + 40.47}{2} - \frac{89.44 + 67.95}{2} = -36.89$

$\hat{Var}(\hat{L}) = \hat{\sigma}^2 \sum_{i=1}^{4} \frac{c_i^2}{n_i} = 6.1 \sum_{i=1}^{4} \left(\frac{1}{4}\right) \frac{1}{10} = \frac{6.1}{10} = .61$

$\hat{SD}(\hat{L}) = \sqrt{.61} = .781$

Allowance: $S \times \hat{SD}(\hat{L}) = 2.93237 \times .781 = 2.2902$

CI: -36.89 ± 2.29 OR $(-39.2, -34.6)$

Scheffé 95% CI's for pairwise differences

Allowance: $S \times \hat{SD}(\bar{Y}_{i.} - \bar{Y}_{j.}) = 2.93237 \times 1.1045 = 3.2388$

CI: Estimate $\pm$ 3.24

Rust Inhibitors. Simultaneous 95% Confidence Intervals by Scheffé's Method

Parameter	Contrast	Estimate	SE	Confidence Interval
D_1	$\mu_A - \mu_B$	-46.30	1.11	(-49.5, -43.1)
D_2	$\mu_A - \mu_C$	-24.81	1.11	(-28.0, -21.6)
D_3	$\mu_A - \mu_D$	2.67	1.11	(-0.6, 5.91)
D_4	$\mu_B - \mu_C$	21.49	1.11	(18.2, 24.7)
D_5	$\mu_B - \mu_D$	48.97	1.11	(45.7, 52.2)
D_6	$\mu_C - \mu_D$	27.48	1.11	(24.2, 30.7)
L	$\frac{\mu_A+\mu_D}{2} - \frac{\mu_B+\mu_C}{2}$	-36.89	.78	(-39.2, -34.6)

We are 95% confident in all seven confidence intervals as a group. Only one comparison is not significant at level $\alpha = .05$, $D_3 = \mu_A - \mu_D$. Inhibitor B is best, followed by C, and then A & D are tied for third — A appears slightly better, but the difference is not significant

The true average score of the best two inhibitors (B, C) is between 35 and 39 points higher than the true average of the worst two inhibitors (A, D).