Lecture 5 Friday January 20

1) Lecture notes pp. 20 & 21
2) HW01 Explain solution to Problem 4
3) HW01 Show answers to other problems

Application of Rules to Find $E(\bar{X})$ & Var$(\bar{X})$

$E(X_1)$

Suppose that $X_1, X_2, \ldots, X_n$ are a random sample from a distribution with mean μ and standard deviation σ. Then

$$E(\bar{X}) = E\left(\frac{\Sigma X_i}{n}\right) = E\left[\sum_{i=1}^{n} \overbrace{\frac{1}{n}}^{a_i} X_i\right] \underset{L}{=} \sum_{i=1}^{n} \frac{1}{n} E(X_i)$$

Rule for EV of linear combination, p.18

$$= \sum_{i=1}^{n}\left(\frac{1}{n}\mu\right) = n \cdot \left(\frac{1}{n}\mu\right) = \mu.$$

So, $E(\bar{X}) = \mu$

$$\text{Var}(\bar{X}) = \text{Var}\left(\frac{\Sigma X_i}{n}\right) = \text{Var}\left(\sum_{i=1}^{n} \frac{1}{n} X_i\right) = \sum_{i=1}^{n} \frac{1}{n^2}(\text{Var } X_i)$$

$$= \sum_{i=1}^{n}\left(\frac{1}{n^2}\sigma^2\right) = n\left(\frac{\sigma^2}{n^2}\right) = \frac{\sigma^2}{n}$$

$$SD(\bar{X}) = \sqrt{\text{Var}(\bar{X})} = \frac{\sigma}{\sqrt{n}}$$

Distribution of the sample mean $\bar{X}$

Suppose we draw a random sample of size n from a population with expected value μ and standard deviation σ.

Let $\bar{X}$ be the mean of the *sample*. Then:

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1 The mean of the distribution of $\bar{X}$ is equal to μ.

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2 The standard deviation of the distribution of $\bar{X}$ is equal to $\sigma/\sqrt{n}$.

3 If n is large, then the distribution of $\bar{X}$ is approximately normal (Central Limit Theorem), and if the distribution of the X's is normal, the distribution of $\bar{X}$ is exactly normal for any sample size n.
In either case, we use the following as if it were exactly true:

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim \mathcal{N}(0, 1).$$

z-score for Xbar

Concept. The statistic Xbar, before drawing the sample, is a random variable, so it has a probability distribution; we call it the "sampling distribution" of Xbar.

<u>Practical point</u> The bigger the sample size, the more accurate is $\bar{X}$ as an estimator of μ.

HW01 Problem #4 (Students who missed this question on HW01 may rework this question and re-submit it w/ HW02, for full credit toward HW01)

Use either the Central Limit Theorem (CLT) or the Law of Averages in your solution (but not the exact binomial distribution).

Start of a solution involving CLT:

Note that both Scenarios (i) & (ii) involve the event you get at least 60% Heads in a sequence of tosses of a fair coin, with (i) $n=100$ tosses or (ii) $n=1000$.

Consider (i). Let $Y_i = \begin{cases} 1 & i^{th} \text{ toss is Head} \\ 0 & i^{th} \text{ toss is Tails} \end{cases}$

then the proportion of Heads is $\frac{1}{100}\sum_{i=1}^{100} Y_i = \bar{Y}$ ← How?

and $E(\bar{Y}) = .5$, $Var(\bar{Y}) = \frac{\sigma}{\sqrt{100}} = \frac{.5(1-.5)}{\sqrt{100}}$

Now apply the CLT to get the approx. prob. that $\bar{Y} > .6$ // Consider (ii) next.