STA 6857—Regression with Autocorrelated Errors & Transfer Function Modeling (§5.5 & 5.6)

Outline

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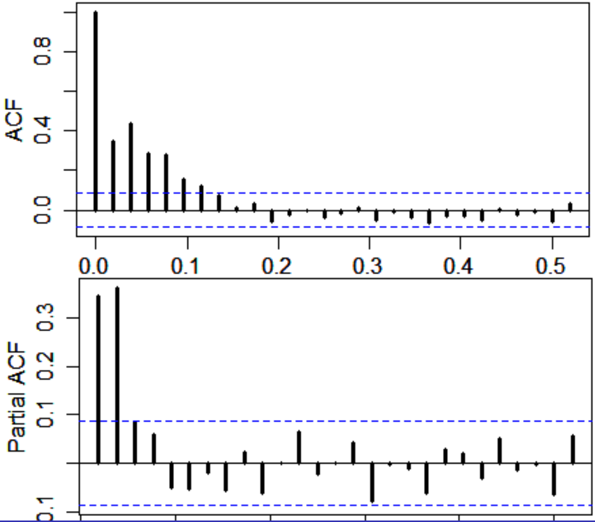
ACF and PACF of Residuals

```
> acf(fit$resid)
> pacf(fit$resid)
```



ACF and PACF of Residuals

```
> (fit2<-ar.ols(fit$resid, aic=F,order=2 ))

Call:
ar.ols(x = fit$resid, aic = F, order.max = 2)

Coefficients:
     1        2
0.2205   0.3625

Intercept: -0.002895 (0.2472)

Order selected 2  sigma^2 estimated as  30.92
```

New Fit

```
> Mort<-filter(mort, c(1,-.2205,-.3625),sides=1)[3:508]
> Trend<-filter(trend, c(1,-.2205,-.3625),sides=1)[3:508]
> Temp<-filter(temp, c(1,-.2205,-.3625),sides=1)[3:508]
> Temp2<-filter(temp2, c(1,-.2205,-.3625),sides=1)[3:508]
> Part<-filter(part, c(1,-.2205,-.3625),sides=1)[3:508]
>
> (fit3 = lm(Mort~ Trend + Temp + Temp2 + Part))

Call:
lm(formula = Mort ~ Trend + Temp + Temp2 + Part)

Coefficients:
(Intercept)        Trend         Temp        Temp2         Part
   34.83550     -0.02777     -0.19616      0.01676      0.22901
```

Summary of Old Fit

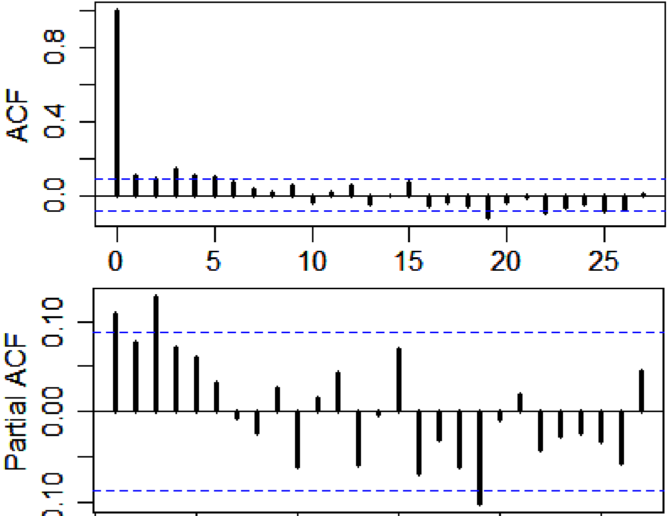
```
> summary(fit)
Call:
lm(formula = mort ~ trend + temp + temp2 + part, na.action = NULL)

Residuals:
     Min       1Q   Median       3Q      Max
-19.0760  -4.2153  -0.4878   3.7435  29.2448

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 81.592238   1.102148   74.03  < 2e-16 ***
trend       -0.026844   0.001942  -13.82  < 2e-16 ***
temp        -0.472469   0.031622  -14.94  < 2e-16 ***
temp2        0.022588   0.002827    7.99 9.26e-15 ***
part         0.255350   0.018857   13.54  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6.385 on 503 degrees of freedom
Multiple R-Squared: 0.5954,     Adjusted R-squared: 0.5922
```

Summary of New Fit

```
> summary(fit3)
Call:
lm(formula = Mort ~ Trend + Temp + Temp2 + Part)

Residuals:
     Min       1Q   Median       3Q      Max
-17.4256  -3.4915  -0.3200   3.0912  17.9067

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 34.835498   0.672217  51.822  < 2e-16 ***
Trend       -0.027775   0.003861  -7.193 2.32e-12 ***
Temp        -0.196162   0.038710  -5.067 5.68e-07 ***
Temp2        0.016758   0.002210   7.582 1.66e-13 ***
Part         0.229008   0.022589  10.138  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 5.281 on 501 degrees of freedom
Multiple R-Squared: 0.3068,     Adjusted R-squared: 0.3012
```

ACF and PACF of Residuals

```
> acf(fit3$resid,lwd=3)
> pacf(fit3$resid,lwd=3)
```



Alternative Method — gls

```
> library(nlme)
> (fit.gls = gls(mort~trend + temp + temp2 + part, correlation=corA
Generalized least squares fit by maximum likelihood
  Model: mort ~ trend + temp + temp2 + part
  Data: NULL
  Log-likelihood: -1549.037

Coefficients:
(Intercept)       trend        temp       temp2        part
87.63747477 -0.02915079 -0.01880909  0.01542466  0.15437383

Correlation Structure: ARMA(2,0)
 Formula: ~1
 Parameter estimate(s):
     Phi1      Phi2
0.3848530 0.4326282
Degrees of freedom: 508 total; 503 residual
Residual standard error: 7.699336
```

ACF plot of residuals showed strong correlation however, so may need some tweaking.

Outline

Transfer Function Modeling

This is a time domain solution to the lagged regression problem we considered earlier using spectral analysis.

It is often easier to fit a transfer function model in the spectral domain as before than in the time domain.

Recall the problem: We wish to find the filter $\alpha(B)$ such that

$$y_t = \sum_{j=0}^{\infty} \alpha_j x_{t-j} + \eta_t = \alpha(B) x_t + \eta_t$$

Modeling $\alpha(B)$

Box and Jenkings considered the following model for α:

$$\alpha(B) = \frac{\delta(B)B^d}{\omega(B)}$$

where

$$\omega(B) = 1 - \omega_1 B - \omega_2 B^2 - \cdots - \omega_r B^r$$

and

$$\delta(B) = \delta_0 + \delta_1 B + \cdots \delta_s B^s$$

Estimation of the parameters is somewhat involved and you are referred to the text for the details.