About the Exam

This worksheet illustrates the types of problems you should expect to see on the upcoming Midterm on **October 15**. However, none of these problems will be repeated on the midterm. It is up to *you* to work through the problems if you choose to, so don't ask the instructor or the TA to solve them for you. You will learn the material much better when solving the problems by yourself. The more difficult problems would be accompanied by hints on the actual exam. Solutions to the problems on the midterm will be provided some time after you have taken to the exam. The midterm may cover material from the following sources:

- Lectures up to October 5th
- Chapters 1, 2, and 3 of the textbook
- Homeworks 1, 2, 3, and 4

You may bring a single $8.5 \times 11''$ sheet of paper with your notes written on both sides of the paper. You will have 50 minutes to take the exam with time being promptly called at 1:40. You may bring a calculator, though you shouldn't need one. I will provide blank white paper for you to write your solutions, but may bring your own paper if you like. **Study hard!**

∘ **1** If $m_t = \sum_{k=0}^{p} c_k t^k$, $t \in \mathbb{Z}$, show that ∇m_t is a polynomial of degree $p-1$ in t and hence that $\nabla^{p+1} m_t = 0$.

∘ **2** Let $Z_t \sim$ iid $\mathcal{N}(0, \sigma^2)$, $t \in \mathbb{Z}$ and $a, b, c \in \mathbb{R}$. Which of the following processes are stationary? For each stationary process specify the mean and autocovariance function.

(a) $X_t = a + bZ_t + cZ_{t-1}$ (b) $X_t = a + bZ_0$ (c) $X_t = Z_1 \cos(ct) + Z_2 \sin(ct)$
(d) $X_t = Z_0 \cos(ct)$ (e) $X_t = Z_t \cos(ct) + Z_{t-1} \sin(ct)$ (f) $X_t = Z_t Z_{t-1}$

∘ **3** Let $\{Y_t\}$ be a stationary process with mean zero and let a and b be constants.

(a) If $X_t = a + bt + s_t + Y_t$ where s_t is a seasonal component with period 12, show that $\nabla\nabla_{12} X_t = (1-B)(1-B^{12})X_t$ is stationary.

(b) If $X_t = (a + bt)s_t + Y_t$ where s_t is again a seasonal component with period 12, show that $\nabla_{12}^2 X_t = (1- B^{12})(1- B^{12})X_t$ is stationary.

∘ **4** Let $x_t = a + bt + y_t$, where $\{y_t, t = 0, \pm 1, \ldots\}$ is an independent and identically distributed sequence of random variables with mean 0 and variance σ^2, and a and b are constants. Define

$$W_t = \frac{1}{2q+1} \sum_{j=-q}^{q} x_{t+j}$$

Compute the mean and autocovariance function of $\{W_t\}$. Notice that although $\{W_t\}$ is not stationary, its autocovariance function does not depend on t. Plot the autocorrelation function $\rho(h)$.

∘ **5** Which, if any, of the following functions defined on the integers is the autocovariance function of a stationary time series?

(a) $f(h) = \begin{cases} 1 & \text{if } h = 0 \\ 1/h & \text{otherwise} \end{cases}$ **(b)** $f(h) = (-1)^{|h|}$ **(c)** $f(h) = 1 + \cos(\frac{\pi h}{2}) + \cos(\frac{\pi h}{4})$

(d) $f(h) = 1 + \cos(\frac{\pi h}{2}) - \cos(\frac{\pi h}{4})$ **(e)** $f(h) = \begin{cases} 1 & \text{if } h = 0 \\ .4 & \text{if } h = \pm 1 \\ 0 & \text{otherwise} \end{cases}$ **(f)** $f(h) = \begin{cases} 1 & \text{if } h = 0 \\ .6 & \text{if } h = \pm 1 \\ 0 & \text{otherwise} \end{cases}$

◦ **6** Let $\{S_t, t = 0, 1, \ldots\}$ be the random walk with constant drift μ, defined by $S_0 = 0$ and $S_t = \mu + S_{t-1} + x_t, t = 1, 2, \ldots$ where $x_1, x_2, \ldots$ are iid $(0, \sigma^2)$. Compute the mean of S_t and the autocovariance function of the process $\{S_t\}$. Show that $\{\nabla S_t\}$ is stationary and compute its mean and autocovariance function.

◦ **7** Give a general solution to the recursion equation

$$u_n = u_{n-1} + u_{n-2} - u_{n-3}$$

◦ **8** Find the range of α such that the AR(2) process

$$x_t = x_{t-1} + \alpha x_{t-2} + w_t$$

is stationary (w_t is white noise). Also, find the ACF for the model with $\alpha = -\frac{1}{2}$.

◦ **9** Specify p, d, q, P, D, and Q in the following SARIMA$(p,d,q)\times(P,D,Q)$ model

$$(1 - .5B)(1 - B^{12})(1 - B)x_t = (1 + B)(1 + 2B^{12})w_t$$

◦ **10** Describe the Box-Jenkins methodology to model selection.

◦ **11** Is following ARMA model causal; is it invertible?

$$x_t = .4x_{t-1} + .45x_{t-2} + w_t + 2w_{t-1} + .25w_{t-2}$$

◦ **12** List five different methods for estimating the AR coefficient in an AR(1) model.

◦ **13** List five different smoothing techniques and describe one of them.

◦ **14** Let Y_t be iid Bernoulli(p) random variables, i.e.

$$Y_t = \begin{cases} 1, & \text{w.p. } p \\ 0, & \text{w.p. } 1-p \end{cases}$$

and define $Z_{2t} = Y_t$ and $Z_{2t+1} = Z_{2t}$. Is Z_t mean stationary? Is Z_t covariance stationary? Is Z_t strictly stationary?