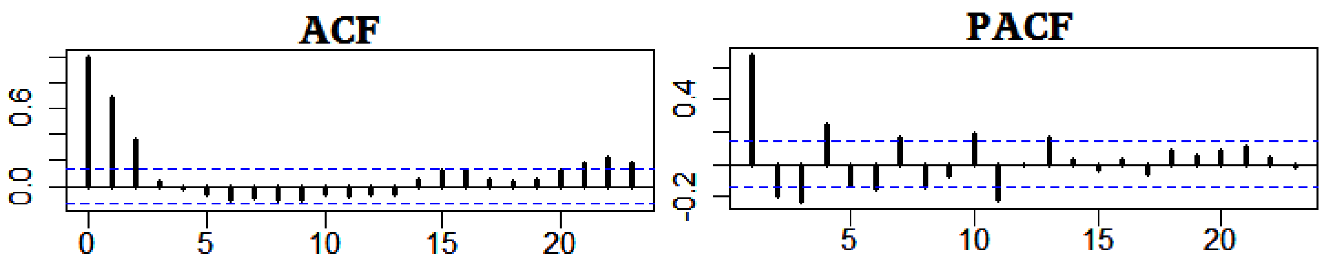
STA 6857 Midterm Exam

Please turn off your cell phones. You have 50 minutes to take the exam.
Relative points values are provided next to each problem. **Good Luck!!** NAME:____________

- **1 (5 points)** If $\{x_t\}$ and $\{y_t\}$ are uncorrelated stationary sequences, i.e. if x_s and y_t are uncorrelated for every s and t, show that $\{x_t + y_t\}$ is stationary and compute its autocovariance function in terms of the autocovariance functions of $\{x_t\}$ and $\{y_t\}$.

- **2 (5 points)** Let $w_t \sim$ iid $\mathcal{N}(0,1)$ and define $x_t = w_t w_{t-1}$. Show that x_t is a white noise sequence. This provides an example of a *dependent* white noise sequence. Two bonus points for rigorously proving x_t and x_{t+1} are not independent.

- **3 (5 points)** A time series of length 200 was generated from a ARMA(p,q) model, and the acf and pacf plots are provided below. Using the Box-Jenkins approach to model selection, what would you estimate p and q to be?



- **4 (8 points)** Write down the general form of a SARIMA$(1,2,3)\times(4,5,6)_{12}$ model with all of its parameters written out. For example, an ARIMA(1,1,1) is written out as $(1-\phi B)\nabla x_t = (1+\theta B)w_t$ where $w_t \sim \text{WN}(0,\sigma^2)$. If you include σ^2, how many parameters are there in a general SARIMA$(p,d,q)\times(P,D,Q)_{12}$ model?

- **5 (10 points)** Given the average maximum temperature in Gainesville during the month of July is 91 °F and the maximum temperature during July follows an AR(1) process with AR parameter $\phi = .5$, what would you predict the maximum temperature to be on the 4$^{\text{th}}$ of July if the temperature on July 2$^{\text{nd}}$ is 95 °F? And what would you predict the temperature to be on July 31$^{\text{st}}$? (Give your predictions based on the best mean square predictor.)

- **6 (5 points)** If $x_t = w_t + \sum_{i=1}^{p} \phi_i x_{t-i}$, where $\{w_t\} \sim \text{WN}(0,\sigma^2)$ and such that w_t is uncorrelated with $\{x_j, j < t\}$ for each t, use the **prediction equations** to show that the best linear mean square predictor of x_{n+1} based on $x_1, \ldots, x_n$ (with $n \geq p$) is

$$\widehat{x}_{n+1} = \sum_{i=1}^{p} \phi_i x_{n+1-i}$$

- **7 (12 points)** Suppose

$$4u_n = 4u_{n-1} - u_{n-2} \qquad (\star)$$

(a) Give the general solution to the difference equation $(\star)$.
(b) Give an exact solution to $(\star)$ given the initial conditions $u_0 = 1$ and $u_1 = \frac{4}{5}$.
(c) Write down an ARMA model whose autocorrelation function satisfies $\rho(k) = u_k$.

#	Points	Score
1	5	
2	5	
3	5	
4	8	

#	Points	Score
5	10	
6	5	
7	12	
Σ	50	