- **Article (2 points)** Find an interesting article in your field that involves one or multiple time series, and print the title, abstract and citation of the article.

- Answers vary.

- **#2.1 (2 points)** Let Z_t, where t is even, be a sequence of independent random variables defined as $Z_t = +1$ or -1 with equal probability of 1/2, and $Z_t = Z_{t-1}$ if t is odd, where t is an integer.

  (a) Is the process first order stationary in distribution?

  (b) Is it second order stationary in distribution?

- (a) Since Z_t has the same distribution for all t, it is first order stationary in distribution.

  (b) Z_t is not second order stationary in distribution. In fact, it is not even second order weakly stationary since, for example, $\operatorname{corr}(Z_0, Z_1) = 1$ but $\operatorname{corr}(Z_1, Z_2) = 0$.

- **#2.3 (2 points)** Prove or disprove that the following process is covariance stationary:

  (a) $Z_t = A\sin(2\pi t + \theta)$, where A is a constant, and θ is a random variable that is uniformly distributed on $[0, 2\pi]$;

  (b) $Z_t = A\sin(2\pi t + \theta)$, where A is a random variable with zero mean and unit variance, and θ is a constant.

  (c) $Z_t = (-1)^t A$, where A is a random variable with zero mean and unit variance.

- (a) This part is not graded.

  (b) This part is not graded.

  (c) Note that $\operatorname{cov}(Z_t, Z_{t+h}) = \operatorname{cov}((-1)^t A, (-1)^{t+h} A) = (-1)^{2t+h}\operatorname{var}(A) = (-1)^h$ which is free of t. Therefore Z_t is covariance stationary.

- **iid⇒ strongly stationary (2 points)** Prove that a time series of iid random variables is strongly stationary.

- Denote the CDF of the iid random variables, $\{Z_t\}$, by $F(x)$. Then both $\{Z_{t_1}, \ldots, Z_{t_n}\}$ and $\{Z_{t_1+h}, \ldots, Z_{t_n+h}\}$ both have the same joint CDF written as

$$F(x_1, \ldots, x_n) = \prod_{i=1}^{n} F(x_i)$$

  which implies strict stationarity.

- **$\boldsymbol{\alpha_i = \beta_i}$ (2 points)** From slide 6 on Jan 11, show that $\beta_i = \alpha_i$.

- Like on slide 5, after taking the derivative and setting it equal to zero, we have

$$\gamma_i = \beta_1\gamma_{i-1} + \beta_2\gamma_{i-2} + \cdots + \beta_k\gamma_{i-k+1}$$

  Since the α's solve the same recursion, i.e.

$$\gamma_i = \alpha_1\gamma_{i-1} + \alpha_2\gamma_{i-2} + \cdots + \alpha_k\gamma_{i-k+1}$$

  We conclude $\alpha_i = \beta_i$ (for $i = 1, \ldots, k-1$).

- **uncorrelated exercise** From slide 6 on Jan 11, show that $Z_t - \widehat{Z}_t$ and $Z_{t+k} - \widehat{Z}_{t+k}$ are uncorrelated with each of $Z_t, \ldots, Z_{t+k-1}$.

- This problem is not graded.