Orthosis Data Set: Example of Functional ANOVA

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An orthotic is an orthopedic device that is applied externally to limb or body. The main purpose can be to provide support, stability, prevention of deformity from getting worse or replacement of lost function. There are a large variety of orthosis available depending on the diagnosis and physical needs of the individual. The data were acquired and computed by Dr. Amarantini David and Dr. Martin Luc (Laboratoire Sport et Performance Motrice, EA 597, UFRAPS, Grenoble University, France). The purpose of recording such data was the interest to better understand the processes underlying movement generation under various levels of an externally applied moment to the knee. In this experiment, stepping-in-place was a relevant task to investigate how muscle redundancy could be appropriately used to cope with an external perturbation while complying with the mechanical requirements related either to balance control and/or minimum energy expenditure. Seven young male volunteers wore a spring-loaded orthosis of adjustable stiffness under four experimental conditions: a control condition (without orthosis), an orthosis condition (with the orthosis only), and two conditions (spring1, spring2) in which stepping in place was perturbed by fitting a spring-loaded orthosis onto the right knee joint. The experimental session included 10 trials of 20 seconds under each experimental condition for each subject. Data sampling started 5 seconds after the onset of stepping, and lasted for 10 seconds for each trial. So, anticipatory and joint movements induced by the initiation of the movement were not sampled. For each of the seven subjects, ten stepping cycles of data were analyzed under each experimental condition. The resultant moment at the knee is derived by means of body segment kinematics recorded with a sampling frequency of 200 Hz. We refer to Cahouet *et al.* (2002) for further details on how the data were recorded and how the resultant moment was computed.

For each stepping-in-place replication, the resultant moment was computed at 256 time points equally spaced and scaled so that a time interval corresponds to an individual gait cycle. A typical moment observation is therefore a one-dimensional function of time t, where $t \in [0, 1]$. The data set consists in 280 separate runs and involves the seven subjects over four described experimental conditions, replicated ten times for each subject. Figure 2 shows the available data set; typical moment plots over gait cycles. Since the purpose of the experiment was to understand how a subject can cope with the external perturbation, we

Figure 1: Examples of Knee Orthosis.

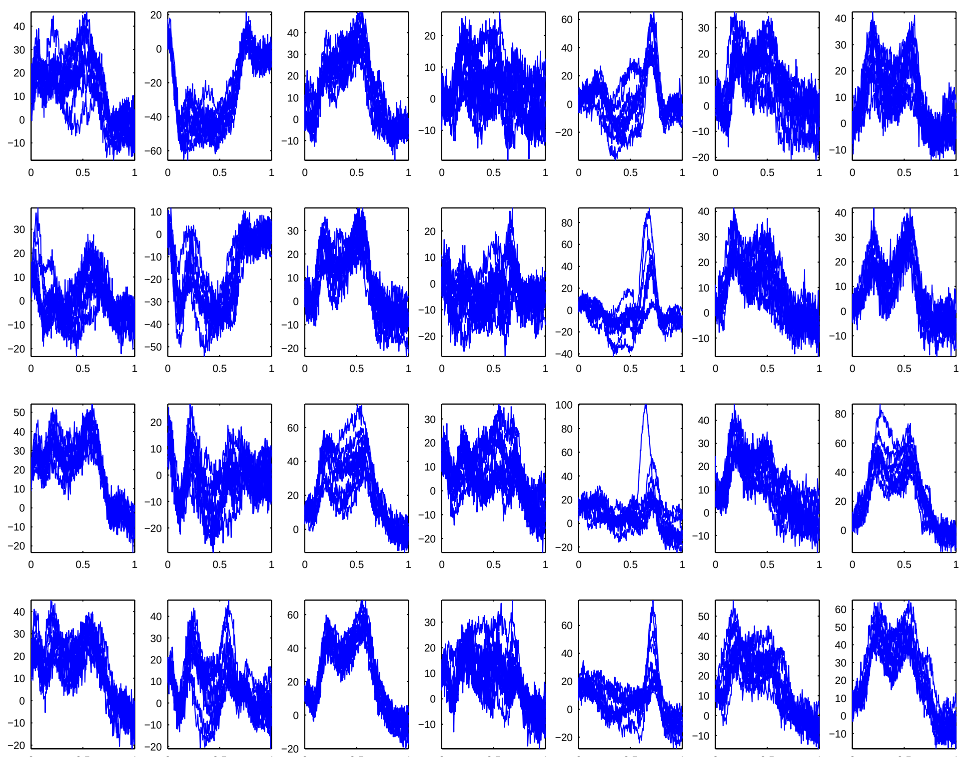

Figure 2: Orthosis data set: panels in rows correspond to *Treatments* while the panels in columns correspond to *Subjects*.

need to quantify the ways in which the individual mean cross-sectional functions differ over the various conditions. We model the data as arising from a fixed-effects FANOVA model with 2 qualitative factors and 10 replications for each level combination, and the purpose is now to show that substantial insight can be gained by applying directly the proposed functional hypothesis testing procedures. The appropriate model for the available data set is a block design, written as

$$dY_{ijk}(t) = m_{ij}(t)\ dt + \epsilon\ dW_{ijk}(t),\ i = 1, \dots, I;\ j = 1, \dots, J;\ k = 1, \dots, K;\ t \in [0, 1],$$

with

$$m_{ij}(t) = m_0 + \mu(t) + \alpha_i + \gamma_i(t) + \beta_j + \delta_j(t),\ i = 1, \dots, I;\ j = 1, \dots, J;\ t \in [0, 1],$$

where i is the condition index, j is the subject index, k is the replication index and t is the time variable. Subjects in the above model are naturally considered as block effects; subjects obviously differ but the researchers are not interested in their differences. Treating subjects as blocks allows us to make inference about the treatments of interest more precise since variability due to block effects is excluded (in this example, we have $I = 4$ treatments). Therefore, given the standard identifiability constraints, we can now apply the proposed FANOVA methodology on the averaged model

$$d\bar{Y}_{i\cdot\cdot}(t) = m_i(t)\ dt + \eta\ dW_{i\cdot\cdot}(t),\ i = 1, \dots, I;\ t \in [0, 1],$$

with

$$m_i(t) = m_0 + \mu(t) + \alpha_i + \gamma_i(t),\ i = 1, \dots, I;\ t \in [0, 1],$$

where $\eta = \epsilon/\sqrt{JK}$.

Since the number of available detail levels in the wavelet decomposition is seven ($\log_2(256) - 1$), we adopted $j(s) = 4$ and $j_\eta = 6$. That is, we expect that a smooth function can be well described within the first four coarsest levels, the treatment effect functions require up to six levels and the seventh level contains the noise. For each treatment, we used the median of absolute deviation of the empirical wavelet coefficients of the empirical estimators of the corresponding treatment effect at the highest resolution level divided by 0.6745 as an estimate of $\epsilon/\sqrt{JK}$. The final estimate of $\epsilon/\sqrt{JK}$ is the average over all treatments and found to be 0.3586. This is a sensible procedure since one can assume that the functional components $\mu(t)$ and $\gamma_i(t)$ of the averaged model are sufficiently smooth and that their presence at the finest levels of detail in wavelet decompositions is minimal. We also found that our tests are robust with respect to the selection of wavelets used. In this analysis we have used the compactly supported *Coiflet 18-tap filter* mother wavelet, motivated by their excellent compromise in smoothness, compactness, almost-symmetry, and good approximation characteristics.

We first test the hypothesis $H_0 : \mu(t) = 0$ versus $H_1 : \mu(t) \in \mathcal{F}(\rho)$ (for some $\rho > 0$). An empirical estimator of overall mean $m_0 + \mu(t)$ is shown in Figure 3a. It is therefore reasonable to test $H_0 : \mu(t) = 0$ with the version of the proposed test given in the paper for $p \geq 2$. The null hypothesis is rejected, the value of the statistic $T(j(s))$ being 12602.54 with p-value essentially equal to 0 (the 5%-level critical value is 0.3686). The test statistic

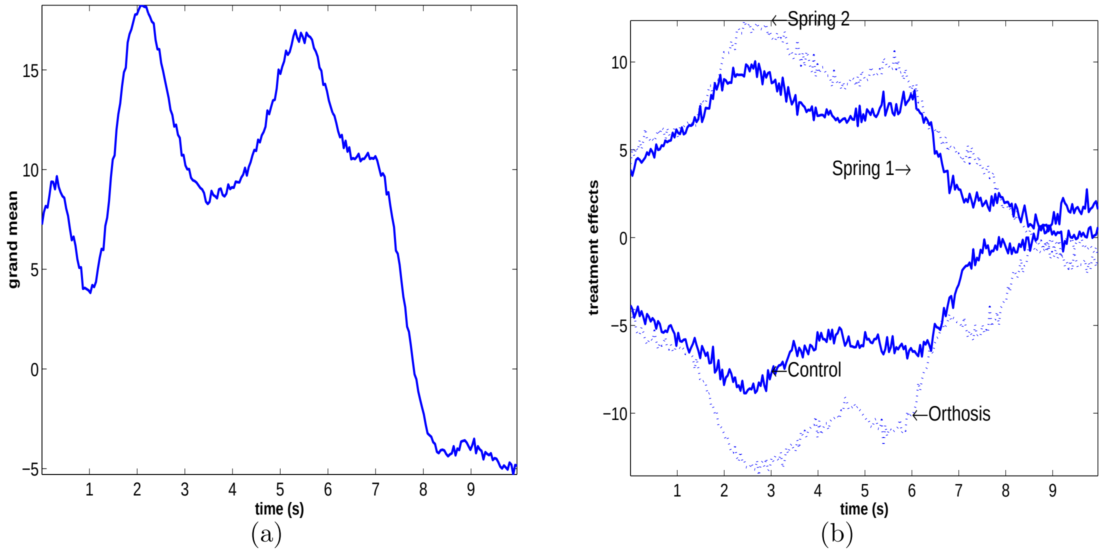

Figure 3: Panel (a) depicts the time domain empirical estimator of the grand mean, $m_0+\mu(t)$. The noise is suppressed by averaging, but still of size $\epsilon/\sqrt{IJK}$; Panel (b) depicts empirical estimators of the treatment effects of interest. Constant and functional components α_i and $\gamma_i(t)$ $(i = 1, \ldots, 4)$ are not separated.

$T(j(s)) + Q(j(s))$ for testing the null hypothesis of equality of treatment effects $H_0 : \gamma_i = 0$ $(i = 1, ..., 4)$ takes the value 13531.30 and also leads to reject H_0 with a p-value essentially equal to 0 (the 5%-level critical value is 309.62).

The researchers were also interested in testing the contrasts: (i) Control and Orthosis functional treatment effects are equal $(H_0 : \gamma_1(t) = \gamma_2(t))$, and (ii) Spring 1 and Spring 2 functional treatment effects are equal $(H_0 : \gamma_3(t) = \gamma_4(t))$. By inspecting the corresponding empirical estimators, shown in Figure 3b, it is again reasonable to test these hypotheses using $p \geq 2$. Both of these hypotheses are not rejected, with $T(j(s))$ equal to 158.95 and 134.03 respectively. The 5%-level critical values are 309.62 for both tests and the p-values are 0.157 and 0.198, respectively. One can therefore say that under Control and Orthosis conditions the subjects behave similarly, the same being true under Spring 1 and Spring 2 conditions.

By inspecting the empirical estimators of treatments, shown in Figure 3b, one could hypothesize that the contrast $(\gamma_1(t) + \gamma_2(t)) - (\gamma_3(t) + \gamma_4(t))$ is significant. In terms of the application, this means that the sum of Control and Orthosis differ from the sum of Spring 1 and Spring 2. The corresponding null hypothesis is highly rejected; the p-value is 0.0103, with a test statistics $T(j(s))$ equal to 46276.42 compared to a 5%-level critical value equal to 619.25.

In conclusion, we have found that all null hypotheses in this study had been rejected except the contrasts $\gamma_1 - \gamma_2 \equiv 0$ and $\gamma_3 - \gamma_4 \equiv 0$, which were not rejected at level $\alpha = 5\%$.

This supports the fact that individuals adjust their posture similarly under perturbations of similar nature.